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التمرين 4)

$\lim_{x\to 0^+} x\ln^3(x)$ • (1

$$\begin{aligned}\lim_{x\to 0^+} x\ln^3(x) &= \lim_{x\to 0^+}\left(\sqrt[3]{x}\ln(x)\right)^3\\ &= \lim_{x\to 0^+}\left(\sqrt[3]{x}\ln\left(\sqrt[3]{x}\right)^3\right)^3\\ &= \lim_{x\to 0^+} 27\left(\sqrt[3]{x}\ln\sqrt[3]{x}\right)^3\\ &= 0\end{aligned}$$

(لأن $\lim_{\alpha\to 0^+}\alpha\ln\alpha = 0$)

$\lim_{x\to 0^+}\sqrt{x}\ln(x)$ • (2

$$\begin{aligned}\lim_{x\to 0^+}\sqrt{x}\ln(x) &= \lim_{x\to 0^+}\sqrt{x}\ln\left(\sqrt{x}^2\right)\\ &= \lim_{x\to 0^+} 2\sqrt{x}\ln\left(\sqrt{x}\right)\\ &= 0\end{aligned}$$

$\lim_{x\to 0^+} x\ln(\sqrt{x})$ • (3

$$\begin{aligned}\lim_{x\to 0^+} x\ln(\sqrt{x}) &= \lim_{x\to 0^+} x\ln\left(x^{\frac{1}{2}}\right)\\ &= \lim_{x\to 0^+}\frac{1}{2}x\ln(x)\\ &= 0\end{aligned}$$

$\lim_{x\to 0^+}\frac{\ln(1-x)}{x}$ • (4

$$\begin{aligned}\lim_{x\to 0^+}\frac{\ln(1-x)}{x} &= \lim_{x\to 0^+}\frac{\ln(1-x)-\ln(1-0)}{x-0}\\ &= -1\end{aligned}$$

نهاية معدل تغير الدالة $f: x\mapsto \ln(1-x)$ على يمين الصفر أي $f'_d(0)$. لكل $x\prec 1$ لدينا :

$f'(x)=\frac{-1}{1-x}$ أي $f'_d(0)=-1$

$\lim_{x\to -2}\frac{\ln(x+3)}{x+2}$ • (5

$$\lim_{x\to -2}\frac{\ln(x+3)}{x+2}=\lim_{x\to -2}\frac{\ln(1+(x+2))}{x+2}=\lim_{h\to 0}\frac{\ln(1+h)}{h}=1$$

$\lim_{x\to +\infty} x\ln(1+\frac{1}{x})$ • (6

$$\lim_{x\to +\infty} x\ln(1+\frac{1}{x})=\lim_{x\to +\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}=\lim_{h\to 0^+}\frac{\ln(1+h)}{h}=1$$

$\lim_{x\to e^+}\frac{\ln(\ln x)}{x-e}$ • (7

$$\lim_{x\to e^+}\frac{\ln(\ln x)}{x-e}=\lim_{x\to e^+}\frac{\ln(\ln x)-\ln(\ln e)}{x-e}=\frac{1}{e}$$

نهاية معدل تغير الدالة $f:x\mapsto \ln(\ln(x))$ على يمين العدد e . لكل $x\succ 1$ لدينا

$f'(x)=\frac{1}{\ln(x)}\times\frac{1}{x}$ إذن $f_d'(e)=\frac{1}{\ln(e)}\times\frac{1}{e}=\frac{1}{e}$

$\lim_{x\to 0^+}\frac{\ln(\cos x)}{x^2}$ • (8

$$\lim_{x\to 0^+}\frac{\ln(\cos x)}{x^2}=\lim_{x\to 0^+}\frac{\ln(\cos x)}{\cos x-1}\times\frac{\cos x-1}{x^2}=\lim_{h\to 1^-}\frac{\ln(h)}{h-1}\times\lim_{x\to 0^+}\frac{\cos x-1}{x^2}=1\times-\frac{1}{2}=-\frac{1}{2}$$

10) • $\lim_{x\to+\infty} \frac{\ln^2(x)}{x}$

$$\begin{aligned}
\lim_{x\to+\infty} \frac{\ln^2(x)}{x} &= \lim_{x\to+\infty} \left(\frac{\ln(x)}{\sqrt{x}}\right)^2 \\
&= \lim_{x\to+\infty} \left(\frac{\ln \sqrt{x}^{2}}{\sqrt{x}}\right)^2 \\
&= \lim_{x\to+\infty} \left(2\frac{\ln \sqrt{x}}{\sqrt{x}}\right)^2 \\
&= \lim_{x\to+\infty} 4\left(\frac{\ln \sqrt{x}}{\sqrt{x}}\right)^2 \\
&= 0
\end{aligned}$$

(لأن $\lim_{\alpha\to+\infty} \frac{\ln\alpha}{\alpha}=0$ ، يمكنك وضع $\alpha=\sqrt{x}$ ، $x\to+\infty$ ، $\alpha\to+\infty$)

11) • $\lim_{x\to+\infty} \ln(x) - x^2 + x$

$$\begin{aligned}
\lim_{x\to+\infty} \ln(x) - x^2 + x &= \lim_{x\to+\infty} x^2\left(\frac{\ln x}{x^2} - 1 + \frac{1}{x}\right) \\
&= \lim_{x\to+\infty} x^2\left(\frac{\ln x}{x^2} - 1 + \frac{1}{x}\right) \\
&= \lim_{x\to+\infty} x^2\left(\frac{1}{x}\left(\frac{\ln x}{x}\right) - 1 + \frac{1}{x}\right) \\
&= -\infty
\end{aligned}$$

12) • $\lim_{x\to-\infty} x + \ln(x^2+3x)$

$$\lim_{x\to-\infty} x+\ln(x^2+3x) = \lim_{x\to-\infty} x\left(1+\frac{\ln(x^2+3x)}{x}\right)$$
$$= \lim_{x\to-\infty} x\left(1+\frac{\ln(x^2)}{x}\right)$$
$$= \lim_{x\to-\infty} x\left(1+\frac{\ln(-x)^2}{x}\right)$$
$$= \lim_{x\to-\infty} x\left(1-2\frac{\ln(-x)}{-x}\right)$$
$$= -\infty$$

(لأن $\lim_{x\to-\infty}\frac{\ln(-x)}{-x} = \lim_{h\to+\infty}\frac{\ln(h)}{h} = 0$)

13) • $\lim_{x\to+\infty}\frac{\ln(x^2-x+1)}{x}$

$$\lim_{x\to+\infty}\frac{\ln(x^2-x+1)}{x} = \lim_{x\to+\infty}\frac{\ln\left(x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)\right)}{x}$$
$$= \lim_{x\to+\infty} 2\frac{\ln(x)}{x}+\frac{\ln\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}{x}$$
$$= 0$$

14) • $\lim_{x\to-\infty}\frac{\ln(x^2+1)}{x^3+4}$

$$\lim_{x\to-\infty}\frac{\ln(x^2+1)}{x^3+4} = \lim_{x\to-\infty}\frac{\ln(x^2+1)}{(x^2+1)}\times\frac{(x^2+1)}{x^3+4}$$
$$= \lim_{x\to-\infty}\frac{\ln(x^2+1)}{(x^2+1)}\times\frac{1}{x}$$
$$= \lim_{h\to+\infty}\frac{\ln h}{h}\times\lim_{x\to-\infty}\frac{1}{x}$$
$$= 0$$

15) • $\lim_{x\to+00}\frac{x-\ln(x)}{x+\ln(x)}$

$$\lim_{x\to+\infty}\frac{x-\ln(x)}{x+\ln(x)}=\lim_{x\to+\infty}\frac{x\left(1-\frac{\ln(x)}{x}\right)}{x\left(1+\frac{\ln(x)}{x}\right)}$$
$$=\lim_{x\to+\infty}\frac{\left(1-\frac{\ln(x)}{x}\right)}{\left(1+\frac{\ln(x)}{x}\right)}$$
$$=1$$

$\lim_{x\to0^+}\frac{x-\ln(x)}{x+\ln(x)}$ • (16

$$\lim_{x\to0^+}\frac{x-\ln(x)}{x+\ln(x)}=\lim_{x\to0^+}\frac{\ln(x)\left(\frac{x}{\ln(x)}-1\right)}{\ln(x)\left(\frac{x}{\ln(x)}+1\right)}$$
$$=\lim_{x\to0^+}\frac{\left(\frac{x}{\ln(x)}-1\right)}{\left(\frac{x}{\ln(x)}+1\right)}$$
$$=-1$$

$\lim_{x\to+\infty}x(\ln(x)-\ln(x+1))$ • (17

$$\lim_{x\to+\infty}x(\ln(x)-\ln(x+1))=\lim_{x\to+\infty}x\ln\left(\frac{x}{x+1}\right)$$
$$=\lim_{x\to+\infty}-\frac{x}{x+1}\,\frac{\ln\left(\frac{x}{x+1}\right)}{\left(\frac{x}{x+1}\right)-1}$$
$$=\left(\lim_{x\to+\infty}-\frac{x}{x+1}\right)\left(\lim_{h\to1}\frac{\ln(h)}{h-1}\right)$$
$$=-1$$

$\lim_{x\to0^+}\frac{\ln(1+\sqrt{x})}{x\ln(x)}$ • (18

$$\lim_{x\to 0^+} \frac{\ln(1+\sqrt{x})}{x\ln(x)} = \lim_{x\to 0^+} \frac{\ln(1+\sqrt{x})}{\sqrt{x}\left(\sqrt{x}\ln(x)\right)}$$

$$= \lim_{x\to 0^+} \frac{\ln(1+\sqrt{x})}{\sqrt{x}} \times \frac{1}{2\sqrt{x}\ln\sqrt{x}}$$

$$= \lim_{h\to 0^+} \frac{1}{2}\frac{\ln(1+h)}{h} \times \frac{1}{h\ln(h)}$$

$$= -\infty$$

(لأن $\lim_{x\to 0^+} \frac{\ln(1+h)}{h} = 1$ و $\lim_{x\to 0^+} h\ln(h) = 0^-$)

$\lim_{x\to +\infty} \sqrt{x} - (\ln(x))^2$ • (19

$$\lim_{x\to +\infty} \sqrt{x} - (\ln(x))^2 = \lim_{x\to +\infty} \sqrt{x}\left(1 - \frac{(\ln(x))^2}{\sqrt{x}}\right)$$

$$= \lim_{x\to +\infty} \left(\sqrt[4]{x}\right)^2 \left(1 - 16\left(\frac{\ln\sqrt[4]{x}}{\sqrt[4]{x}}\right)^2\right)$$

$$= \lim_{h\to +\infty} h^2\left(1 - 16\times\left(\frac{\ln(h)}{h}\right)^2\right)$$

$$= +\infty$$

$\lim_{x\to ?} \frac{x}{x-1} + \ln(\sqrt{x-1})$ • (20