

$$\begin{aligned}
 & \cdot (M_2(\mathbb{R}), \times) \quad E \quad - \quad - (2) \\
 & : \quad E \quad M(c, d) \quad M(a, b) \\
 & M(a, b) \times M(c, d) = (aI + bJ) \times (cI + dJ) = acI + (bc + ad)J + bdJ^2 \\
 & : \quad J^2 = -I : \\
 & M(a, b) \times M(c, d) = (ac - bd)I + (bc + ad)J = M(ac - bd, bc + ad) \\
 & \cdot (M_2(\mathbb{R}), \times) \quad E \\
 & \cdot (E^*, \times) \quad (C^*, \times) \quad f \quad - \\
 & z \times z' = (ac - bd) + i(bc + ad) : \quad C^* \quad z' = c + id \quad z = a + ib \\
 & f(z \times z') = M(ac - bd, bc + ad) : \\
 & \cdot f(z \times z') = M(a, b) \times M(c, d) = f(z) \times f(z') : \quad - \\
 & \cdot (E^*, \times) \quad (C^*, \times) \quad f \\
 & : \quad E \quad (I, J) \\
 & (\forall M \in E^*) ; (\exists!(a, b) \in \mathbb{R}^2 - \{(0, 0)\}) / M = aI + bJ \\
 & : \quad aI + bJ = M(a, b) = f(a + ib) : \\
 & \cdot (\forall M \in E^*) ; (\exists! z = a + ib \in C^*) / M = f(a + ib) = f(z) \\
 & \cdot \quad E^* \quad C^* \quad f \\
 & \cdot \quad (E, +, \times) \quad - (3) \\
 & \cdot \quad (E, +) \quad (E, +, \cdot) \\
 & (E^*, \times) \quad (C^*, \times) \quad f \quad (C^*, \times) \\
 & \cdot \quad (E^*, \times)
 \end{aligned}$$

$$\begin{aligned}
 & \cdot (M_2(\mathbb{R}), +, \cdot) \quad (E, +, \cdot) \quad - \quad - (1) \\
 & \cdot E \neq \emptyset : \quad (M(0, 0) = O) \quad E \quad O \quad - \\
 & : \quad E \quad M(c, d) \quad M(a, b) \quad \mathbb{R}^2 \quad (\lambda, \mu) \quad - \\
 & \lambda M(a, b) + \mu M(c, d) = M(\lambda a + \mu c, \lambda b + \mu d) \\
 & \cdot \lambda M(a, b) + \mu M(c, d) \in E : \\
 & \cdot (M_2(\mathbb{R}), +, \cdot) \quad (E, +, \cdot) \\
 & : \quad E \quad M(a, b) \quad - \\
 & M(a, b) = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & \sqrt{3}b \\ -\frac{1}{\sqrt{3}}b & 0 \end{pmatrix} = aI + bJ \\
 & \cdot (E, +, \cdot) \quad (I, J) \\
 & \cdot \quad (I, J) \quad - \\
 & \cdot \alpha I + \beta J = O : \quad \mathbb{R}^2 \quad (\alpha, \beta) \\
 & \left(\begin{matrix} \alpha & \sqrt{3}\beta \\ -\frac{1}{\sqrt{3}}\beta & \alpha \end{matrix} \right) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} : \quad \alpha I + \beta J = M(\alpha, \beta) : \\
 & \cdot \alpha = \beta = 0 : \quad \alpha = \sqrt{3}\beta = -\frac{1}{\sqrt{3}}\beta = 0 : \\
 & \cdot E \quad (I, J) \\
 & \cdot (E, +, \cdot) \quad (I, J)
 \end{aligned}$$

$$\Delta = (a - \bar{a})^2 - 2i(a - \bar{a}) + i^2 = (a - \bar{a} - i)^2 :$$

: \mathbb{C}

$$\Delta = (a - \bar{a} - i)^2 : (G)$$

$$z_1 = \frac{i - a - \bar{a} + a - \bar{a} - i}{2i} = -\frac{2\bar{a}}{2i} = i\bar{a}$$

$$z_2 = \frac{i - a - \bar{a} - a + \bar{a} + i}{2i} = \frac{2(i - a)}{2i} = 1 + ia$$

$$1 + ia = a \quad a = i\bar{a} \quad (G) \quad a \quad - (2)$$

$$1 + ia = a \Leftrightarrow a = \frac{1}{1 - i} = \frac{1 + i}{2} :$$

$$a = i\bar{a} \Leftrightarrow \operatorname{Re}(a) + i \operatorname{Im}(a) = \operatorname{Im}(a) + i \operatorname{Re}(a) \Leftrightarrow \operatorname{Re}(a) = \operatorname{Im}(a)$$

$$\operatorname{Re}(a) = \operatorname{Im}(a) \quad (G) \quad a :$$

$$\operatorname{Re}(a) \neq \operatorname{Im}(a) \quad a \in \mathbb{C} \quad Z = \frac{(1 + ia) - a}{(i\bar{a}) - a} : \quad - (1 - II)$$

$$\bar{Z} = \frac{\overline{(1 + ia) - a}}{\overline{(i\bar{a}) - a}} = \frac{1 - i\bar{a} - \bar{a}}{-i\bar{a} - \bar{a}} = \frac{i + \bar{a} - i\bar{a}}{a - i\bar{a}} = \frac{(i - 1)\bar{a} - i}{i\bar{a} - a} :$$

$$Z = \bar{Z} \quad Z \in \mathbb{R} \quad C(1 + ia) \quad B(i\bar{a}) \quad A(a)$$

$$(1 + ia) - a = (i - 1)\bar{a} - i : \quad \frac{(1 + ia) - a}{i\bar{a} - a} = \frac{(i - 1)\bar{a} - i}{i\bar{a} - a} :$$

$$\operatorname{Im}(a) = \frac{1}{2} : \quad 2i \operatorname{Im}(a) = \frac{(1 + i)^2}{2} : \quad a - \bar{a} = \frac{1 + i}{1 - i} :$$

$$(1 + i)^2 = 2i :$$

$M_2(\mathbb{R})$

$E \quad M_2(\mathbb{R})$

. E

$(E, +, \times)$

$$J \times X^3 = I : \quad E \quad - (4)$$

$$(\forall X \in E^*); J \times X^3 = I \Leftrightarrow f^{-1}(J \times X^3) = f^{-1}(I) \Leftrightarrow f^{-1}(J) \times f^{-1}(X^3) = f^{-1}(I)$$

$$f^{-1}(X^3) = (f^{-1}(X))^3 \quad f^{-1}(J) = i \quad f^{-1}(I) = 1 :$$

$$f^{-1}(X^3) = z^3 : \quad z = x + iy \quad X = M(x, y)$$

$$(z - i) \times (z^2 + iz - 1) = 0 \quad z^3 = -i = i^3 \quad i \times z^3 = 1$$

$$z_2 = \frac{\sqrt{3} - i}{2} \quad z_1 = -\frac{\sqrt{3} + i}{2} \quad z_0 = i :$$

$$E \quad J \times X^3 = I$$

$$S = \left\{ M(0, 1) = J; M\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right); M\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \right\}$$

$$(G) : iz^2 + (a + \bar{a} - i)z - \bar{a} - ia\bar{a} = 0 : \quad \mathbb{C} \quad - I$$

$$(G) \quad - (1)$$

$$\Delta = (a + \bar{a} - i)^2 + 4i(\bar{a} + ia\bar{a}) = (a + \bar{a})^2 - 2i(a + \bar{a}) - 1 + 4i\bar{a} - 4a\bar{a}$$

$$= a^2 + 2a\bar{a} + \bar{a}^2 - 2ia - 2i\bar{a} + 4i\bar{a} - 4a\bar{a} - 1$$

$$= a^2 - 2a\bar{a} + \bar{a}^2 - 2i(a - \bar{a}) - 1$$

■

(E): $35u - 96v = 1$: \mathbb{Z}^2 -I

(E) (11,4) $35 \times 11 - 96 \times 4 = 385 - 384 = 1$: - (1)

$35(u - 11) - 96(v - 4) = 0$: (E) (u,v) - (2)

$96 \mid 35(u - 11)$: (*) $35(u - 11) = 96(v - 4)$:

$96 \mid (u - 11)$: $96 \wedge 35 = 1$:

$u = 11 + 96k$ / $k \in \mathbb{Z}$:

$v = 4 + 35k$: (*)

$S = \{ (11 + 96k, 4 + 35k) / k \in \mathbb{Z} \}$: (E)

(F): $x^{35} \equiv 2 \pmod{97}$: \mathbb{N} -II

(F) x - (1)

$p^2 \leq 97$ p 97 -

97	p	q	r	97
	2	48	1	:
	3	32	1	.
	5	19	2	.
	7	13	5	.

$p^2 \leq 97$

$d = x \wedge 97$: -

$d = 97$ $d = 1$: 97

$\text{Im}(a) \neq \frac{1}{2}$: - (2)

$R_1(B) = B' \Leftrightarrow z_{B'} - z_A = e^{-i\frac{\pi}{2}}(z_b - z_A) \Leftrightarrow b' - a = -i(\bar{a} - a)$: -

$b' = a - i^2 \bar{a} + ia = (1+i)a + \bar{a}$:

$R_2(C) = C' \Leftrightarrow z_{C'} - z_A = e^{i\frac{\pi}{2}}(z_c - z_A) \Leftrightarrow c' - a = i(1+ia-a)$

$c' = a + i - a - ia = i(1-a)$:

$c' = i(1-a)$ $b' = (1+i)a + \bar{a}$:

$c' - b' = i - ia - a - ia - \bar{a} = i(1-2a) - (a + \bar{a})$: -

$z_E = \frac{z_B + z_C}{2} = \frac{1+ia+i\bar{a}}{2} = \frac{1+i(a+\bar{a})}{2}$: [BC] E

$z_E - z_A = \frac{1+i(a+\bar{a})}{2} - a = \frac{1-2a+i(a+\bar{a})}{2}$:

$\frac{c' - b'}{z_E - z_A} = 2i$: $2i(z_E - z_A) = c' - b'$:

$\frac{c' - b'}{z_E - z_A} \in i\mathbb{R}$: $(AE) \perp (B'C')$

$B'C' = 2AE$: $\frac{B'C'}{AE} = \frac{|c' - b'|}{|z_E - z_A|} = \frac{|c' - b'|}{|z_E - z_A|} = |2i| = 2$:

■
 $f(x) = 2x - e^{-x^2} : \mathbb{R}_+ \rightarrow \mathbb{R}$

$\lim_{x \rightarrow +\infty} (f(x) - 2x) : - - (1)$

$\lim_{x \rightarrow +\infty} (f(x) - 2x) = \lim_{x \rightarrow +\infty} -e^{-x^2} = - \lim_{t \rightarrow +\infty} e^{-t} = 0 :$

$y = 2x : +\infty (C)$

$f'(x) -$

$(\forall x \in \mathbb{R}_+) ; f'(x) = (2x)' - (-x^2)' e^{-x^2} = 2 + 2xe^{-x^2} = 2(1 + xe^{-x^2})$

$((\forall x \in \mathbb{R}_+) ; xe^{-x^2} \geq 0 :) (\forall x \in \mathbb{R}_+) ; f'(x) > 0 :$

$\mathbb{R}_+ \quad f$

x	0	$+\infty$
f'(x)		+
f	-1	$+\infty$

$(\lim_{x \rightarrow +\infty} 2x = +\infty \text{ , } \lim_{x \rightarrow +\infty} e^{-x^2} = 0) \Rightarrow \lim_{x \rightarrow +\infty} f(x) = +\infty$

$\mathbb{R}_+ \quad f \quad \mathbb{R}_+ \quad f -$

$J = f(\mathbb{R}_+) = [f(0); \lim_{x \rightarrow +\infty} f(x)[= [-1; +\infty[$

$f(\alpha) = 0 : \mathbb{R}_+ \quad \alpha \quad 0 \in J$

$0 < \alpha < 1 \quad f(1) = 2 - \frac{1}{e} > 0 \quad f(0) = -1 < 0 :$

$x \quad 97 \quad 97 = x \wedge 97 : \quad d = 97$

$x^{35} \equiv 2 [97] \quad x^{35} \equiv 0 [97] : \quad x^{35} \quad 97 :$

$97 \quad x \quad d = 1$

$x^{97-1} \equiv 1 [97] : \quad x \wedge 97 = 1 \quad 97 -$

$x^{96} \equiv 1 [97] :$

$x^{35 \times 11} \equiv 2^{11} [97] : \quad x^{35} \equiv 2 [97] \Rightarrow (x^{35})^{11} \equiv 2^{11} [97] : -$

$x^{(1+96 \times 4)} \equiv 2^{11} [97] : \quad 35 \times 11 = 1 + 96 \times 4 :$

$x \equiv x^{(1+99 \times 4)} [97] : \quad (\quad) x^{96} \equiv 1 [97] :$

$x \equiv 2^{11} [97] :$

$x^{35} \equiv 2^{11 \times 35} [97] : \quad x \equiv 2^{11} [97] : \quad x \quad - (2)$

$(\quad) 2 \wedge 97 = 1 \Rightarrow 2^{96} \equiv 1 [97] \quad 35 \times 11 = 1 + 96 \times 4 :$

$(F) \quad x \quad x^{35} \equiv 2 [97] : \quad 2^{11 \times 35} \equiv 2 [97] :$

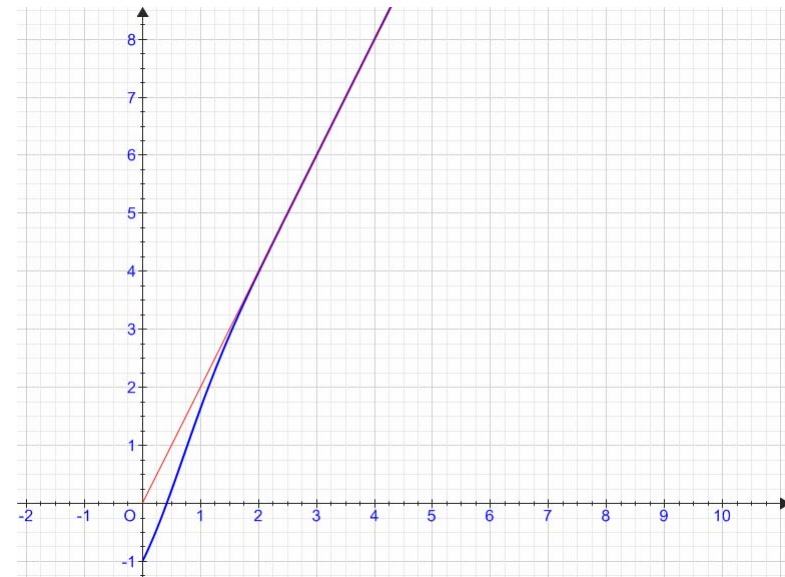
$(F) \quad x \equiv 2^{11} [97] \quad - (3)$

$2^{11} \equiv 11 [97] : \quad 2^{11} = 2048 = 21 \times 97 + 11 :$

$x \equiv 2^{11} [97] \Leftrightarrow x \equiv 11 [97] \Leftrightarrow x = 11 + 97k / k \in \mathbb{N} :$

\mathbb{R}_+^* x $[0; x]$ F
 $F(x) - F(0) = x \cdot F'(c) :]0; x[$ c
 $\int_0^x e^{-t^2} dt - 0 = x \cdot e^{-c^2} :$
 $(\forall x \in \mathbb{R}_+^*) ; (\exists c \in]0; x[) / \frac{1}{x} \int_0^x e^{-t^2} dt = e^{-c^2} :$
 $(\exists c \in]0; 1[) / \int_0^1 e^{-t^2} dt = e^{-c^2} : x = 1 -$
 $\int_0^1 e^{-t^2} dt < 1 : 0 < c < 1 \Rightarrow e^{-c^2} < e^{-0^2} = 1 :$
 $(\forall x \in \mathbb{R}_+) ; g(x) = \int_0^x f(t) dt : - (2)$
 $(\forall x \in \mathbb{R}_+) ; \int_0^x f(t) dt = \int_0^x (2t) dt - \int_0^x e^{-t^2} dt = [t^2]_0^x - \int_0^x e^{-t^2} dt = g(x) :$
 $\mathbb{R}_+ \quad x \mapsto \int_0^x f(t) dt : \mathbb{R}_+ \quad f -$
 $(\forall x \in \mathbb{R}_+) ; \left(\int_0^x f(t) dt \right)' = f(x) :$
 $(\forall x \in \mathbb{R}_+) ; g'(x) = f(x) \quad \mathbb{R}_+ \quad g$
 $] \alpha; 1[\quad g \quad (\forall x \in] \alpha; 1[) ; g'(x) = f(x) > 0 : -$
 $(\forall t \in [0; \alpha]) ; f(t) < 0 \Rightarrow g(\alpha) = \int_0^\alpha f(t) dt < 0 :$
 $\int_0^1 e^{-t^2} dt < 1 \Rightarrow g(1) = 1^2 - \int_0^1 e^{-t^2} dt > 0$
 $g(\beta) = 0 :] \alpha; 1[\quad \beta$

$[0; 1]$ $f(x)$ -
 $0 < \alpha < 1 : \alpha \quad [0; 1] \quad f$
 $(\forall x \in] \alpha; 1]) ; f(x) > 0 \quad (\forall x \in [0; \alpha]) ; f(x) < 0$
 $(C) \quad - (2)$



$\mathbb{R}_+ \quad g \quad \varphi \quad - II$
 $g(x) = x^2 - \int_0^x e^{-t^2} dt \quad \begin{cases} \varphi(x) = \frac{1}{x} \int_0^x e^{-t^2} dt ; x > 0 \\ \varphi(0) = 0 \end{cases}$
 $\mathbb{R}_+^* \quad \mathbb{R}_+ \quad F : u \mapsto \int_0^u e^{-t^2} dt \quad - (1)$
 $(\forall x \in \mathbb{R}_+^*) ; F'(x) = e^{-x^2}$

$$(\forall x \in \mathbb{R}_+^*); \varphi'(x) = (e^{-x^2})' + \left(\frac{2}{x}\right)' \int_0^x t^2 e^{-t^2} dt + \frac{2}{x} \left(\int_0^x t^2 e^{-t^2} dt\right) :$$

$$(\forall x \in \mathbb{R}_+^*); \varphi'(x) = -2xe^{-x^2} - \frac{2}{x^2} \int_0^x t^2 e^{-t^2} dt + \frac{2}{x} x^2 e^{-x^2} :$$

$$(\forall x \in \mathbb{R}_+^*); \varphi'(x) = -\frac{2}{x^2} \int_0^x t^2 e^{-t^2} dt :$$

$$\mathbb{R}_+ \quad \varphi) [0;1] \quad \varphi \quad -$$

$$((\forall x \in \mathbb{R}_+^*); \varphi'(x) < 0 :$$

$$\cdot \varphi([0;1]) = [\varphi(1); \varphi(0)] = [\varphi(1); 1] :$$

$$[\varphi(1); 1] \subset [0;1] : \quad 0 < \varphi(1) = \int_0^1 e^{-t^2} dt < 1 :$$

$$\cdot \varphi([0;1]) \subset [0;1] :$$

$$(\forall x \in \mathbb{R}_+); \int_0^x t^2 e^{-t^2} dt \leq \frac{x^3}{3} : \quad - \quad - (4)$$

$$(\forall t \in [0;x]); t^2 e^{-t^2} \leq t^2 : \quad e^{-t^2} \leq 1 : \quad [0;x] \quad t$$

$$(\forall x \in \mathbb{R}_+); \int_0^x t^2 e^{-t^2} dt \leq \int_0^x t^2 dt = \left[\frac{t^3}{3}\right]_0^x = \frac{x^3}{3} :$$

$$(\forall x \in \mathbb{R}_+^*); |\varphi'(x)| = \frac{2}{x^2} \int_0^x t^2 e^{-t^2} dt : \quad -$$

$$\frac{2}{x^2} \quad (\forall x \in \mathbb{R}_+); \int_0^x t^2 e^{-t^2} dt \leq \frac{x^3}{3} :$$

$$(\forall x \in \mathbb{R}_+^*); |\varphi'(x)| \leq \frac{2}{3} x :$$

$\varphi \quad - \quad - (3)$

$\mathbb{R}_+^* \quad x$

$$e^{-x^2} \leq e^{-t^2} \leq 1 : \quad -x^2 \leq -t^2 \leq 0 : \quad [0;x] \quad t$$

$$\int_0^x e^{-x^2} dt \leq \int_0^x e^{-t^2} dt \leq \int_0^x 1 dt :$$

$$\cdot xe^{-x^2} \leq \int_0^x e^{-t^2} dt \leq x :$$

$$(\forall x \in \mathbb{R}_+^*); e^{-x^2} \leq \varphi(x) = \frac{1}{x} \int_0^x e^{-t^2} dt \leq 1 :$$

$$\lim_{x \rightarrow 0^+} \varphi(x) = 1 = \varphi(0) : \quad \lim_{x \rightarrow 0^+} e^{-x^2} = 1$$

φ

$$(\forall x \in \mathbb{R}_+^*); \varphi(x) = e^{-x^2} + \frac{2}{x} \int_0^x t^2 e^{-t^2} dt : \quad -$$

$$\begin{cases} u'(t) = -2te^{-t^2} \\ v(t) = t \end{cases} : \quad \begin{cases} u(t) = e^{-t^2} \\ v'(t) = 1 \end{cases} :$$

$$\frac{1}{x} \int_0^x e^{-t^2} dt = \left[te^{-t^2}\right]_0^x + 2 \int_0^x t^2 e^{-t^2} dt :$$

$$\cdot \varphi(x) = e^{-x^2} + \frac{2}{x} \int_0^x t^2 e^{-t^2} dt$$

$$\mathbb{R}_+ \quad x \mapsto \int_0^x t^2 e^{-t^2} dt \quad \mathbb{R}_+ \quad t \mapsto t^2 e^{-t^2} \quad -$$

$$(\forall x \in \mathbb{R}_+); \left(\int_0^x t^2 e^{-t^2} dt\right)' = x^2 e^{-x^2} :$$

$$\mathbb{R}_+^* \quad \varphi \quad \mathbb{R}_+^* \quad x \mapsto e^{-x^2} \quad x \mapsto \frac{2}{x} :$$

$$|u_{n+1} - \beta| = |\varphi(u_n) - \varphi(\beta)| :$$

: β u_n

$$|\varphi(u_n) - \varphi(\beta)| \leq \frac{2}{3} |u_n - \beta|$$

$$\cdot |u_{n+1} - \beta| \leq \left(\frac{2}{3}\right)^{n+1} \quad |u_n - \beta| \leq \left(\frac{2}{3}\right)^n$$

$$\cdot (\forall n \in \mathbb{N}) ; |u_n - \beta| \leq \left(\frac{2}{3}\right)^n :$$

$(u_n)_{n \geq 0}$ -

$$\lim_{n \rightarrow +\infty} \left(\frac{2}{3}\right)^n = 0 : \quad 0 < \frac{2}{3} < 1$$

$$(u_n)_{n \geq 0} \quad (\forall n \in \mathbb{N}) ; |u_n - \beta| \leq \left(\frac{2}{3}\right)^n :$$

β

:

$$(x \in]0;1[\Rightarrow \frac{2}{3}x \leq \frac{2}{3} :) . (\forall x \in]0;1[); |\varphi'(x)| \leq \frac{2}{3} :$$

: \mathbb{R}_+^* x -

$$\varphi(x) = x \Leftrightarrow \frac{1}{x} \int_0^x e^{-t^2} dt = x \Leftrightarrow \int_0^x e^{-t^2} dt = x^2 \Leftrightarrow x^2 - \int_0^x e^{-t^2} dt = 0 \Leftrightarrow g(x) = 0$$

$$\cdot (\forall x \in \mathbb{R}_+^*); \varphi(x) = x \Leftrightarrow g(x) = 0 :$$

: $(u_n)_{n \geq 0}$ - (5)

$$\cdot (\forall n \in \mathbb{N}); u_{n+1} = \varphi(u_n) \quad u_0 = \frac{2}{3}$$

$$\cdot (\forall n \in \mathbb{N}); 0 \leq u_n \leq 1 : -$$

$$\cdot 0 \leq u_0 = \frac{2}{3} \leq 1 : \quad n = 0$$

$$0 \leq u_n \leq 1 :$$

$$u_{n+1} = \varphi(u_n) \in [0;1] : \quad \varphi([0;1]) \subset [0;1] :$$

$$\cdot 0 \leq u_{n+1} \leq 1 :$$

$$\cdot (\forall n \in \mathbb{N}); 0 \leq u_n \leq 1 :$$

: $n = 0$ -

$$\cdot |u_0 - \beta| \leq |1 - 0| = 1 = \left(\frac{2}{3}\right)^0 : \quad \beta \in [0;1] \quad u_0 = \frac{2}{3} \in [0;1]$$

$$\cdot |u_n - \beta| \leq \left(\frac{2}{3}\right)^n :$$

$$\cdot \varphi(\beta) = \beta : \quad - \quad - (4) \quad g(\beta) = 0 :$$