

$$z_1^2 = 4(\sqrt{3} + i) :$$

$$z_2 = (\sqrt{3} - 1) + i(\sqrt{3} + 1) = i \left[(\sqrt{3} + 1) - i(\sqrt{3} - 1) \right]$$

$$(1) \quad \cdot z_2 = i \overline{z_1} :$$

$$\sqrt{3} + i = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right) :$$

$$(0,25) \quad \cdot 4(\sqrt{3} + i) = \left[8, \frac{\pi}{6} \right] :$$

$$z_1 = \left[\sqrt{8}, \frac{\pi}{12} \right] = \left[2\sqrt{2}, \frac{\pi}{12} \right] : \quad z_1^2 = 4(\sqrt{3} + i) = \left[8, \frac{\pi}{6} \right] :$$

$$z_2 = i \overline{z_1} = \left[1, \frac{\pi}{2} \right] \times \left[2\sqrt{2}, -\frac{\pi}{12} \right] = \left[2\sqrt{2}, \frac{\pi}{2} - \frac{\pi}{12} \right]$$

$$(1) \quad \cdot z_2 = \left[2\sqrt{2}, \frac{5\pi}{12} \right] :$$

$$\arg\left(\frac{z_2}{z_1}\right) \equiv \arg(z_2) - \arg(z_1) [2\pi] :$$

$$\cdot \arg\left(\frac{z_2}{z_1}\right) \equiv \frac{\pi}{3} [2\pi] : \quad \arg\left(\frac{z_2}{z_1}\right) \equiv \frac{5\pi}{12} - \frac{\pi}{12} [2\pi] :$$

$$\overline{(\overline{OA}, \overline{OB})} \equiv \frac{\pi}{3} [2\pi] : \quad \overline{(\overline{OA}, \overline{OB})} \equiv \arg\left(\frac{z_2}{z_1}\right) [2\pi] :$$

$$OB = OA = 2\sqrt{2} : \quad |z_2| = |z_1| = 2\sqrt{2} :$$

$$(1) \quad \cdot \quad OAB$$

$$(1) : r^2 - 6r + 9 = 0 \quad y'' - 6y' + 9y = 0 : \quad (1)$$

$$\cdot \Delta = (-6)^2 - 4 \times 9 = 0$$

$$: \quad r_0 = \frac{6}{2} = 3 :$$

$$0,75 \quad \cdot y : x \mapsto (Ax + B)e^{3x} / (A, B) \in \mathbb{R}^2$$

$$: \quad \mathbb{R} \quad u : x \mapsto x^2 e^{3x} : \quad (2)$$

$$\forall x \in \mathbb{R} : \begin{cases} u'(x) = (3x^2 + 2x)e^{3x} \\ u''(x) = (9x^2 + 12x + 2)e^{3x} \end{cases}$$

$$u''(x) - 6u'(x) + 9u(x) = [9x^2 + 12x + 2 - 6(3x^2 + 2x) + 9x^2]e^{3x} :$$

$$u''(x) - 6u'(x) + 9u(x) = 2e^{3x} :$$

$$0,75 : \quad u : x \mapsto x^2 e^{3x}$$

$$\cdot (E) : y'' - 6y' + 9y = 2e^{3x}$$

$$z : x \mapsto (x^2 + Ax + B)e^{3x} / (A, B) \in \mathbb{R}^2 : \quad (E)$$

0,5

$$\Delta' = 3(1+i)^2 - 8i = -2i : \quad z^2 - 2\sqrt{3}(1+i)z + 8i = 0 \quad (1)$$

$$: \quad \Delta' = (1-i)^2 :$$

$$0,75 \quad \cdot z_2 = (\sqrt{3} - 1) + i(\sqrt{3} + 1) \quad z_1 = (\sqrt{3} + 1) + i(\sqrt{3} - 1)$$

$$z_1^2 = [(\sqrt{3} + 1) + i(\sqrt{3} - 1)]^2$$

$$= (\sqrt{3} + 1)^2 - (\sqrt{3} - 1)^2 + 2i(\sqrt{3}^2 - 1^2) = 4\sqrt{3} + 4i = 4(\sqrt{3} + i)$$

$$\Omega A^2 - \Omega O^2 = (a-1)^2 + (b+1)^2 + (c-3)^2 - (a^2 + b^2 + c^2) :$$

$$\Omega A^2 - \Omega O^2 = -2a + 1 + 2b + 1 - 6c + 9 :$$

$$\Omega A^2 - \Omega O^2 = 33 \Leftrightarrow -2a + 1 + 2b + 1 - 6c + 9 = 33 :$$

$$2a - 2b + 6c + 22 = 0 :$$

$$(1,25) \quad a - b + 3c = -11 :$$

$$\begin{cases} 11a = -11 \\ b = -a \\ c = 3a \end{cases} : \begin{cases} a - b + 3c = -11 \\ b = -a \\ c = 3a \end{cases} \quad \Omega \quad -$$

$$(0,5) \cdot \Omega(-1, 1, -3) :$$

$$\begin{cases} x = 0 + 1t \\ y = 0 - 1t / t \in \mathbb{R} \\ z = 0 + 3t \end{cases} : (O, \overline{OA}) \quad (OA) \quad - (1)$$

$$(0,5) \cdot (OA) : \begin{cases} x = t \\ y = -t / t \in \mathbb{R} \\ z = 3t \end{cases}$$

$$: \quad \overline{OA} \quad (Q) \quad (Q) \perp (OA) : \quad -$$

$$M(x, y, z) \in (Q) \Leftrightarrow \overline{AM} \cdot \overline{OA} = 0$$

$$\Leftrightarrow (x-1) - (y+1) + 3(z-3) = 0$$

$$\Leftrightarrow x - y + 3z - 11 = 0$$

$$(0,75) \quad (Q) : x - y + 3z - 11 = 0 :$$

$$(P) // (Q) : (Q) \perp (P) \quad \vec{n}(1, -1, 3) \quad ($$

$$(0,25)$$

$$(\Omega A) \perp (Q) : A \quad (S) \quad (Q) \quad - (2)$$

$$(OA) = (\Omega A) : (OA) // (\Omega A) : (OA) \perp (Q) :$$

$$\cdot \Omega \in (OA) :$$

$$\Omega(a, b, c) \in (OA) \Leftrightarrow \exists t \in \mathbb{R} / \begin{cases} a = t \\ b = -t \\ z = 3t \end{cases}$$

$$(0,75) \cdot c = 3a \quad b = -a :$$

$$(r = \Omega A \quad \Omega \quad (S) \quad (Q) \quad -$$

$$OA^2 + \Omega O^2 = \Omega A^2 : \sqrt{33} \quad O \quad \Gamma$$

$$(A \in T \quad OA = \sqrt{33} \quad) \quad 33 + \Omega O^2 = \Omega A^2 :$$

$$\cdot \Omega A^2 - \Omega O^2 = 33 :$$

$$g(x) = \ln(1+x) - x : [0, +\infty[\quad g \quad -I$$

$$\forall x \in [0, +\infty[: g'(x) = (-x)' + (\ln(1+x))' : - (1)$$

$$= -1 + \frac{(1+x)'}{1+x}$$

$$= -1 + \frac{1}{1+x}$$

$$= \frac{-x}{x+1}$$

$$g \quad]0, +\infty[\quad x \quad g'(x) < 0 :$$

$$(0,75) \cdot [0, +\infty[$$

$$: [0, +\infty[\quad g \quad -$$

$$g(0) = \ln 1 = 0 \quad \forall x \in [0, +\infty[: g(x) \leq g(0)$$

$$\cdot \forall x \in [0, +\infty[: g(x) \leq 0 :$$

$$(0,25)$$

(0,5) . $\lim_{x \rightarrow 1^+} f(x) = 1 + \lim_{x \rightarrow 1^+} \ln\left(\frac{x+1}{x-1}\right) = +\infty$:

$\forall x \in D : f'(x) = (x)' + \frac{\left(\frac{x+1}{x-1}\right)'}{\left(\frac{x+1}{x-1}\right)} = 1 + \frac{\left| \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} \right|}{(x-1)^2} \times \frac{x-1}{x+1}$: - (3)

$f'(x) = 1 - \frac{2}{(x-1)(x+1)}$:

(0,75) . $\forall x \in D : f'(x) = \frac{x^2 - 1 - 2}{x^2 - 1} = \frac{x^2 - 3}{x^2 - 1}$:

$\forall x \in D : f'(x) = \frac{(x + \sqrt{3})(x - \sqrt{3})}{x^2 - 1}$: -

$f \quad \forall x \in]1, +\infty[: sg[f'(x)] = sg[x - \sqrt{3}]$:

(0,5) . $]1, \sqrt{3}[$ $[\sqrt{3}, +\infty[$

$\lim_{x \rightarrow \pm\infty} f(x) - x = \lim_{x \rightarrow \pm\infty} \ln\left(\frac{x+1}{x-1}\right) = 0$: - (4)

$\lim_{x \rightarrow \pm\infty} \frac{x+1}{x-1} = 1 \Rightarrow \lim_{x \rightarrow \pm\infty} \ln\left(\frac{x+1}{x-1}\right) = \ln 1 = 0$:

(0,25) . (C) $y = x$ (Δ)

$\forall x \in D : \frac{x+1}{x-1} - 1 = \frac{2}{x-1}$: -

$\forall x \in]-\infty, -1[: \frac{x+1}{x-1} < 1 \quad \forall x \in]1, +\infty[: \frac{x+1}{x-1} > 1$

$\forall x \in]-\infty, -1[: \ln\left(\frac{x+1}{x-1}\right) < \ln 1 = 0 \quad \forall x \in]1, +\infty[: \ln\left(\frac{x+1}{x-1}\right) > \ln 1 = 0$

(0,5)

$\forall x \in]0, +\infty[: g(x) < g(0) : [0, +\infty[$ g (2)

$\forall x \in]0, +\infty[: g(x) < 0$:

$g(x) < 0 \Leftrightarrow \ln(1+x) < x$

$\ln(1+x) > \ln 1 = 0 : \forall x \in]0, +\infty[: 1+x > 1 :$

$]0, +\infty[$ \ln

(0,5) . $\forall x \in]0, +\infty[: 0 < \ln(1+x) < x$:

$f(x) = x + \ln\left(\frac{x+1}{x-1}\right) : f$ -II

$: f$ D (1)

$x \in D \Leftrightarrow \begin{cases} x-1 \neq 0 \\ \frac{x+1}{x-1} > 0 \end{cases} \Leftrightarrow (x+1)(x-1) > 0 \Leftrightarrow x \in]-\infty, -1[\cup]1, +\infty[$

(0,5) . $D =]-\infty, -1[\cup]1, +\infty[$:

0 f - (2)

$\forall x \in D : f(-x) = -x + \ln\left(\frac{-x+1}{-x-1}\right) = -x + \ln\left(\frac{x-1}{x+1}\right)$

$\forall x \in D : \ln\left(\frac{x-1}{x+1}\right) = -\ln\left(\frac{x+1}{x-1}\right) :$

$\forall x \in D : f(-x) = -f(x)$

(0,5) . f

$\lim_{x \rightarrow +\infty} \frac{x+1}{x-1} = 1 \Rightarrow \lim_{x \rightarrow +\infty} \ln\left(\frac{x+1}{x-1}\right) = \ln 1 = 0$: -

$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} x = +\infty$:

$\lim_{x \rightarrow 1^+} \ln\left(\frac{x+1}{x-1}\right) = +\infty : \lim_{x \rightarrow 1^+} \frac{x+1}{x-1} = \lim_{x \rightarrow 1^+} \frac{2}{x-1} = +\infty :$

(0,75) . $(u_n)_{n \geq 2} \quad \forall n \geq 2 : u_n > u_{n+1} :$ - (2)

$(x = \frac{2}{n-1}) \quad \forall n \geq 2 : 0 < \ln\left(1 + \frac{2}{n-1}\right) < \frac{2}{n-1}$

(0,5) . $\forall n \geq 2 : 0 < u_n < \frac{2}{n-1} :$

$\lim_{n \rightarrow +\infty} u_n = 0 : \quad \lim_{n \rightarrow +\infty} \frac{2}{n-1} = 0 \quad \forall n \geq 2 : 0 < u_n < \frac{2}{n-1}$ - (0,5)

(1) : (C) _____ (5)

$\forall x \in]1, +\infty[: f(x) > x \quad \forall x \in]-\infty, -1[: f(x) < x$

$]1, +\infty[\quad (\Delta) \quad (C) \quad]-\infty, -1[$

(0,25) . $(\Delta) \quad (C)$

$\begin{cases} u'(x) = \frac{-2}{x^2-1} : \\ v(x) = x \end{cases} \quad \begin{cases} u(x) = \ln\left(\frac{x+1}{x-1}\right) : \\ v'(x) = 1 \end{cases} - (6)$

$\int_2^4 \ln\left(\frac{x+1}{x-1}\right) dx = \left[x \ln\left(\frac{x+1}{x-1}\right) \right]_2^4 + \int_2^4 \frac{2x}{x^2-1} dx :$

(1,25) $= \left[x \ln\left(\frac{x+1}{x-1}\right) + \ln(x^2-1) \right]_2^4$

$\int_2^4 \ln\left(\frac{x+1}{x-1}\right) dx = 4 \ln\left(\frac{5}{3}\right) + \ln(15) - 3 \ln(3) = 5 \ln 5 - 6 \ln 3 :$

$y = x \quad x = 4 \quad x = 2 \quad (C)$ -

(0,5) . $\int_2^4 [f(x) - x] dx = \int_2^4 \ln\left(\frac{x+1}{x-1}\right) dx = (5 \ln 5 - 6 \ln 3) cm^2 :$

$\forall n \geq 2 : u_n = f(n) - n : \quad (u_n)_{n \geq 2} - III$

$\frac{n+1}{n-1} = \frac{n-1+2}{n-1} = 1 + \frac{2}{n-1} \quad \forall n \geq 2 : u_n = \ln\left(\frac{n+1}{n-1}\right) : - (1)$

(0,25) . $\forall n \geq 2 : u_n = \ln\left(1 + \frac{2}{n-1}\right) :$

$\forall n \geq 2 : \frac{2}{n-1} > \frac{2}{n+1-1} : \quad \forall n \geq 2 : \frac{2}{n-1} > \frac{2}{n} : -$

$\ln\left(1 + \frac{2}{n-1}\right) > \ln\left(1 + \frac{2}{n+1-1}\right) \quad 1 + \frac{2}{n-1} > 1 + \frac{2}{n+1-1} :$

