

الحساب المثلثي

تمرين 1

احسب بدلالة $\sin x$ و $\cos x$:

$$D = \cos\left(\frac{\pi}{2} - x\right), \quad C = \sin\left(\frac{\pi}{6} - x\right), \quad B = \cos\left(\frac{\pi}{4} + x\right), \quad A = \sin\left(\frac{\pi}{4} + x\right)$$

$$F = \sin(2x) - 3\cos\left(\frac{\pi}{6} + x\right), \quad E = 2\cos\left(\frac{\pi}{6} - x\right) + \sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

$$H = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x + \frac{4\pi}{3}\right), \quad G = \cos\left(x + \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3} - x\right) - \sin(x)$$

تمرين 2

$$J = \frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x, \quad I = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x \quad : \cos(x+b) \text{ شكل اكتب على شكل}$$

$$F = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x, \quad E = \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x \quad : \sin(x+b) \text{ شكل اكتب على شكل}$$

تمرين 3

$$H = \cos(7a)\cos(3a) - \sin(7a)\sin(3a), \quad G = \cos(2a)\cos a + \sin(2a)\sin a \quad : \text{بسط ما يلي}$$

$$J = \frac{\sqrt{2}}{2}\cos\left(\frac{a}{2}\right) + \frac{\sqrt{2}}{2}\sin\left(\frac{a}{2}\right), \quad I = \frac{1}{2}\sin(3a) + \frac{\sqrt{3}}{2}\cos(3a)$$

تمرين 4

$$\cos\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right) = \cos^2 x - \frac{1}{4} \quad \blacklozenge \quad \text{ليكن } x \in \mathbb{R}, \text{ بين أن :}$$

$$2\sin^2\left(\frac{\pi}{8} + x\right) = 1 - \frac{\sqrt{2}}{2}(\cos 2x - \sin 2x) \quad \blacklozenge \quad (\sin x + \sin 5x)^2 + (\cos x + \cos 5x)^2 = 4\cos^2 2x \quad \blacklozenge$$

تمرين 5

$$-1 \text{ تحقق أن: } \frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$$

$$-2 \text{ استنتج حساب النسب المثلثية لـ } \frac{\pi}{12}$$

تمرين 6

$$-1 \text{ بين أن: } \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right)$$

$$-2 \text{ استنتج قيمة: } \tan\left(\frac{\pi}{8}\right)$$

تمرين 7

ليكن a و b عددين حقيقيين بحيث : $\cos a = \frac{1}{4}$ و $\sin b = \frac{3}{7}$ ، $a \in \left[0; \frac{\pi}{2}\right]$ و $b \in \left[\frac{\pi}{2}; \pi\right]$

♦ احسب $\sin a$ و $\cos b$ واستنتج حساب : $\cos 2a$ و $\cos 2b$ و $\sin 2a$ و $\sin 2b$

تمرين 8

بين أن : $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$ و $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$ وأن

تمرين 9

أكتب على شكل جداء : $\cos x + \cos 2x$ ، $\sin 3x + \sin 5x$ ، $\cos 3x + \cos 7x$ ، $\sin x - \sin \frac{x}{2}$

تمرين 10

حل في IR المعادلات التالية : $\cos x - \sin x = \sqrt{2}$ ، $\cos x - \sqrt{3} \sin x = 1$ ، $\sqrt{3} \cos x - \sin x = \sqrt{2}$

$\sin x + \cos x = 1$ ، $\sin x + \cos x = \sqrt{2}$ ، $\cos \frac{x}{2} - \sin \frac{x}{2} = -1$

تمرين 11

نعتبر الدالة : $f(x) = \frac{\cos x + \sin x}{\cos x - \sin x}$

-1 حل في IR المعادلة $\cos x - \sin x = 0$

-2 حدد Df

-3 بين أن : $\forall x \in Df \quad f(x) = \frac{1 + \sin 2x}{\cos 2x}$

-4 حل في IR المعادلة $f(x) - \sqrt{3} = 0$

تمرين 12

ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = \sqrt{3} \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right)$

-1 بين أن : $A(x) = 2 \cos\left(2x - \frac{\pi}{6}\right)$

-2 حل في المجال $]-\pi; \pi]$ المعادلة : $A(x) = 1$

تمرين 13

ليكن x عددا حقيقيا، نعتبر التعبير : $A(x) = -2 \cos^2(x) + \sqrt{3} \sin(2x) + 2$

-1 بين أن : $A(x) = 4 \sin(x) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right)$

-2 حل في IR المعادلة : $A(x) = 0$

حلول الحساب المثلثي

تمرين 1

$$A = \sin\left(\frac{\pi}{4} + x\right) = \sin\left(\frac{\pi}{4}\right)\cos x + \cos\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$$

$$B = \cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$$

$$C = \sin\left(\frac{\pi}{6} - x\right) = \sin\left(\frac{\pi}{6}\right)\cos x - \cos\left(\frac{\pi}{6}\right)\sin x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$

$$D = \cos\left(\frac{\pi}{2} - x\right) = \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x = 0 + \sin x = \sin x$$

$$E = 2\cos\left(\frac{\pi}{6} - x\right) + \sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

$$E = 2\left(\cos\left(\frac{\pi}{6}\right)\cos x + \sin\left(\frac{\pi}{6}\right)\sin x\right) + \sqrt{2}\left(\sin\left(\frac{\pi}{4}\right)\cos x - \cos\left(\frac{\pi}{4}\right)\sin x\right)$$

$$E = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)$$

$$E = \sqrt{3}\cos x + \sin x + \cos x - \sin x$$

$$E = (\sqrt{3} + 1)\cos x$$

$$F = \sin(2x) - 3\cos\left(\frac{\pi}{6} + x\right)$$

$$F = 2\sin x\cos x - 3\left(\cos\left(\frac{\pi}{6}\right)\cos x - \sin\left(\frac{\pi}{6}\right)\sin x\right)$$

$$F = 2\sin x\cos x - 3\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$$

$$F = 2\sin x\cos x - \frac{3\sqrt{3}}{2}\cos x + \frac{3}{2}\sin x$$

$$G = \cos\left(x + \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3} - x\right) - \sin(x)$$

$$G = \cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x - \left(\sin\left(\frac{\pi}{3}\right)\cos x - \cos\left(\frac{\pi}{3}\right)\sin x\right) - \sin x$$

$$G = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x - \sin x$$

$$G = \frac{1 - \sqrt{3}}{2}\cos x + \frac{-\sqrt{3} - 1}{2}\sin x$$

$$\begin{cases} \cos(a + b) = \cos a \cos b - \sin a \sin b & \cos(a - b) = \cos a \cos b + \sin a \sin b \\ \sin(a + b) = \sin a \cos b + \cos a \sin b & \sin(a - b) = \sin a \cos b - \cos a \sin b \end{cases}$$

نذكر بالقواعد الأساسية:



$$H = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x + \frac{4\pi}{3}\right)$$

$$H = \cos x + \cos(x)\cos\left(\frac{2\pi}{3}\right) - \sin x\sin\left(\frac{2\pi}{3}\right) + \cos x\cos\left(\frac{4\pi}{3}\right) - \sin x\sin\left(\frac{4\pi}{3}\right)$$

$$H = \cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$$

$$H = 0$$

$$\cos\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} ; \sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} ; \cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} ; \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} ; \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2} ; \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

نذكر بالقيم الخاصة:

تمرين 2

$$I = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \cos\left(\frac{\pi}{3}\right)\cos x + \sin\left(\frac{\pi}{3}\right)\sin x = \cos\left(\frac{\pi}{3} - x\right) = \cos\left(x - \frac{\pi}{3}\right)$$

$$J = \frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = \sin\left(\frac{\pi}{4}\right)\sin(x) + \cos\left(\frac{\pi}{4}\right)\cos x = \cos\left(\frac{\pi}{4}\right)\cos x + \sin\left(\frac{\pi}{4}\right)\sin(x) = \cos\left(x - \frac{\pi}{4}\right)$$

$$E = \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = \cos\left(\frac{\pi}{4}\right)\sin x - \sin\left(\frac{\pi}{4}\right)\cos x = \sin\left(x - \frac{\pi}{4}\right)$$

$$F = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \sin\left(\frac{\pi}{3}\right)\cos x + \cos\left(\frac{\pi}{3}\right)\sin x = \sin\left(x + \frac{\pi}{3}\right)$$

نطبق نفس القواعد السابقة لكن بشكل معكوس.

تمرين 3

$$G = \cos(2a)\cos a + \sin(2a)\sin a = \cos(2a - a) = \cos(a)$$

$$H = \cos(7a)\cos(3a) - \sin(7a)\sin(3a) = \cos(7a + 3a) = \cos(10a)$$

$$I = \frac{1}{2}\sin(3a) + \frac{\sqrt{3}}{2}\cos(3a) = \cos\left(\frac{\pi}{3}\right)\sin(3a) + \sin\left(\frac{\pi}{3}\right)\cos(3a) = \sin\left(3a + \frac{\pi}{3}\right)$$

$$J = \frac{\sqrt{2}}{2}\cos\left(\frac{a}{2}\right) + \frac{\sqrt{2}}{2}\sin\left(\frac{a}{2}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{a}{2}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{a}{2}\right) = \sin\left(\frac{a}{2} + \frac{\pi}{4}\right) = \sin\left(\frac{2a + \pi}{4}\right)$$

$$\begin{aligned} \cos\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right) &= \left(\cos(x)\cos\left(\frac{\pi}{6}\right) - \sin(x)\sin\left(\frac{\pi}{6}\right)\right)\left(\cos(x)\cos\left(\frac{\pi}{6}\right) + \sin(x)\sin\left(\frac{\pi}{6}\right)\right) \\ &= \left(\frac{\sqrt{3}}{2}\cos(x) - \frac{1}{2}\sin(x)\right)\left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right) \\ &= \left(\frac{\sqrt{3}}{2}\cos x\right)^2 - \left(\frac{1}{2}\sin x\right)^2 = \frac{3}{4}\cos^2 x - \frac{1}{4}\sin^2 x \\ &= \frac{3}{4}\cos^2 x - \frac{1}{4}(1 - \cos^2 x) = \frac{3}{4}\cos^2 x - \frac{1}{4} + \frac{1}{4}\cos^2 x \\ &= \cos^2 x - \frac{1}{4} \end{aligned}$$

$$\begin{aligned} (\sin x + \sin 5x)^2 + (\cos x + \cos 5x)^2 &= \sin^2 x + 2\sin x\sin 5x + \sin^2 5x + \cos^2 x - 2\cos x\cos 5x + \cos^2 5x \\ &= 1 + 1 + 2(\sin x\sin 5x - \cos x\cos 5x) \\ &= 2 - 2(\cos x\cos 5x - \sin x\sin 5x) \\ &= 2 - 2\cos(5x - x) = 2 - 2\cos(4x) \\ &= 2 - 2\cos(2 \times 2x) = 2 - 2(2\cos^2(2x) - 1) \\ &= 2 - 4\cos^2(2x) + 2 \\ &= 4\cos^2(2x) \end{aligned}$$

$$\begin{aligned} 1 - \frac{\sqrt{2}}{2}(\cos 2x - \sin 2x) &= 1 - \left(\frac{\sqrt{2}}{2}\cos 2x - \frac{\sqrt{2}}{2}\sin 2x\right) = 1 - \left(\cos\left(\frac{\pi}{4}\right)\cos(2x) - \sin\left(\frac{\pi}{4}\right)\sin(2x)\right) \\ &= 1 - \cos\left(2x + \frac{\pi}{4}\right) = 1 - \cos\left(2\left(x + \frac{\pi}{8}\right)\right) = 2\sin^2\left(x + \frac{\pi}{8}\right) \end{aligned}$$

$$\begin{aligned} \cos(2x) = 2\cos^2 x - 1 = 1 - 2\sin^2 x &\Leftrightarrow \cos(2x) + 1 = 2\cos^2 x \Leftrightarrow 1 - \cos(2x) = 2\sin^2 x \\ \sin(2x) = 2\sin x \cos x & \end{aligned}$$

بذكر بالخصيات:

$$\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$$

1

$$\begin{aligned} \sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} \\ \cos\left(\frac{\pi}{12}\right) &= \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

2

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} - \sqrt{2})^2}{6 - 2} = \frac{8 - 2\sqrt{12}}{4} = \frac{8 - 4\sqrt{3}}{4} = 2 - \sqrt{3}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b} \quad \text{باستعمال الخاصية:} \quad \tan\left(\frac{\pi}{12}\right) \quad \text{: يمكنك أيضا حساب}$$

تمرين 6

$\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{8}\right) \right)$ $= \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{8}\right) + \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{8}\right) \right) = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)$	لدينا: 1
$\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) \quad \text{منه: } \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right)$ $\sin\left(\frac{\pi}{8}\right) = (\sqrt{2} - 1) \cos\left(\frac{\pi}{8}\right) \quad \text{منه: } \sin\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)$	لدينا: 2
<div style="border: 1px solid red; display: inline-block; padding: 5px; margin-right: 10px;"> $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$ </div> أي $\frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \sqrt{2} - 1$ بالتالي:	
: يمثل التمرين طريقة أخرى لحساب قيمة $\tan\left(\frac{\pi}{8}\right)$	

تمرين 7

$b \in \left] \frac{\pi}{2}; \pi \right[$ و $a \in \left[0; \frac{\pi}{2} \right[$ ، $\sin b = \frac{3}{7}$ و $\cos a = \frac{1}{4}$	
نعلم أن: $\sin^2 a + \cos^2 a = 1$ منه: $\sin^2 a + \left(\frac{1}{4}\right)^2 = 1$ منه: $\sin^2 a = 1 - \frac{1}{16} = \frac{15}{16}$	
وبما أن: $a \in \left[0; \frac{\pi}{2} \right[$ فإن: $\sin a > 0$ بالتالي: $\sin a = \frac{\sqrt{15}}{4}$	
نعلم أن: $\sin^2 b + \cos^2 b = 1$ منه: $\left(\frac{3}{7}\right)^2 + \cos^2 b = 1$ منه: $\cos^2 b = 1 - \frac{9}{49} = \frac{40}{49}$	
وبما أن: $b \in \left] \frac{\pi}{2}; \pi \right[$ فإن: $\cos b < 0$ بالتالي: $\cos b = -\frac{\sqrt{40}}{7}$	
$\cos 2a = 2 \cos^2 a - 1 = 2 \times \frac{1}{16} - 1 = \frac{1}{8} - 1 = -\frac{7}{8}$	$\cos 2b = 2 \cos^2 b - 1 = 2 \times \frac{40}{49} - 1 = \frac{80}{49} - 1 = \frac{31}{49}$
$\sin 2a = 2 \sin a \cos a = 2 \times \frac{\sqrt{15}}{4} \times \frac{1}{4} = \frac{\sqrt{15}}{8}$	$\sin 2b = 2 \sin b \cos b = 2 \times \frac{3}{7} \times \frac{\sqrt{40}}{7} = \frac{6\sqrt{40}}{49}$

$$\begin{aligned}\cos(x+y)\cos(x-y) &= (\cos x \cos y - \sin x \sin y)((\cos x \cos y + \sin x \sin y)) \\ &= (\cos x \cos y)^2 - (\sin x \sin y)^2 = \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y \\ &= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y \\ &= \cos^2 x - \sin^2 y\end{aligned}$$

$$\begin{aligned}\sin(x+y)\sin(x-y) &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ &= (\sin x \cos y)^2 - (\cos x \sin y)^2 \\ &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= (1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y) \\ &= \cos^2 y - \cos^2 x \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y \\ &= \cos^2 y - \cos^2 x\end{aligned}$$

$$\begin{aligned}\sin 3x + \sin 5x &= 2 \sin\left(\frac{3x+5x}{2}\right) \cos\left(\frac{3x-5x}{2}\right) \\ &= 2 \sin(4x) \cos(-x)\end{aligned}$$

$$\begin{aligned}\cos x + \cos 2x &= 2 \cos\left(\frac{x+2x}{2}\right) \cos\left(\frac{x-2x}{2}\right) \\ &= 2 \cos\left(\frac{3x}{2}\right) \cos\left(\frac{-x}{2}\right)\end{aligned}$$

$$\begin{aligned}\sin x - \sin \frac{x}{2} &= 2 \cos\left(\frac{x+\frac{x}{2}}{2}\right) \sin\left(\frac{x-\frac{x}{2}}{2}\right) \\ &= 2 \cos\left(\frac{3x}{4}\right) \sin\left(\frac{x}{4}\right)\end{aligned}$$

$$\begin{aligned}\cos 3x - \cos 7x &= -2 \sin\left(\frac{3x+7x}{2}\right) \sin\left(\frac{3x-7x}{2}\right) \\ &= -2 \sin(5x) \sin(-2x)\end{aligned}$$

$$\begin{aligned}\cos a + \cos b &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \cos a - \cos b &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ \sin a + \sin b &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \sin a - \sin b &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)\end{aligned}$$

! تذكر بقواعد التعميل:



المعادلات الموحدة بالتمرين كلها على شكل: $a \cos x + b \sin x = c$ و نرمل: $r = \sqrt{a^2 + b^2}$

لنحل المعادلة: $\cos x - \sqrt{3} \sin x = 1$

لدينا: $r = 2$ منه:

$$\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}\right) \cos x - \sin\left(\frac{\pi}{3}\right) \sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi / k \in Z \text{ ou } x + \frac{\pi}{3} = \frac{-\pi}{3} + 2k\pi / k \in Z$$

$$x = 2k\pi / k \in Z \text{ ou } x = \frac{-2\pi}{3} + 2k\pi / k \in Z$$

$$S = \{2k\pi / k \in Z\} \cup \left\{ \frac{-2\pi}{3} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

لنحل المعادلة: $\cos x - \sin x = \sqrt{2}$

$$\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = 1$$

$$\cos\left(\frac{\pi}{4} + x\right) = 1$$

لدينا: $r = \sqrt{2}$ منه:

$$x + \frac{\pi}{4} = 2k\pi / k \in Z$$

$$x = \frac{-\pi}{4} + 2k\pi / k \in Z$$

$$S = \left\{ \frac{-\pi}{4} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

لنحل المعادلة: $\cos \frac{x}{2} - \sin \frac{x}{2} = -1$

$$\frac{1}{\sqrt{2}} \cos \frac{x}{2} - \frac{1}{\sqrt{2}} \sin \frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4}\right) \cos \frac{x}{2} - \sin\left(\frac{\pi}{4}\right) \sin \frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) = -1$$

لدينا: $r = \sqrt{2}$ منه:

$$\frac{x}{2} + \frac{\pi}{4} = \pi + k\pi / k \in Z$$

$$\frac{x}{2} = \frac{3\pi}{4} + k\pi / k \in Z$$

$$x = \frac{3\pi}{2} + 2k\pi / k \in Z$$

$$S = \left\{ \frac{3\pi}{2} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

لنحل المعادلة: $\sqrt{3} \cos x - \sin x = \sqrt{2}$

لدينا: $r = 2$ منه:

$$\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi / k \in Z \text{ ou } x + \frac{\pi}{4} = \frac{-\pi}{4} + 2k\pi / k \in Z$$

$$x = \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi / k \in Z \text{ ou } x = \frac{-\pi}{3} - \frac{\pi}{4} + 2k\pi / k \in Z$$

$$x = \frac{\pi}{12} + 2k\pi / k \in Z \text{ ou } x = \frac{-7\pi}{12} + 2k\pi / k \in Z$$

$$S = \left\{ \frac{\pi}{12} + 2k\pi / k \in Z \right\} \cup \left\{ \frac{-7\pi}{12} + 2k\pi / k \in Z \right\}$$

<p>لنحل المعادلة: $\sin x + \cos x = 1$</p> <p>لدينا: $r = 2$ منه:</p> $\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$ $\cos\left(\frac{\pi}{4}\right) \cos x + \sin\left(\frac{\pi}{4}\right) \sin x = \frac{\sqrt{2}}{2}$ $\cos\left(\frac{\pi}{4} - x\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$ $\frac{\pi}{4} - x = \frac{\pi}{4} + 2k\pi / k \in Z \text{ ou } \frac{\pi}{4} - x = \frac{-\pi}{4} + 2k\pi / k \in Z$ $x = -2k\pi / k \in Z \text{ ou } x = \frac{\pi}{2} + 2k\pi / k \in Z$ <p>بالتالي: $S = \{-2k\pi / k \in Z\} \cup \left\{\frac{\pi}{2} + 2k\pi / k \in Z\right\}$</p>	<p>لنحل المعادلة: $\sin x + \cos x = \sqrt{2}$</p> $\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = 1$ $\cos\left(\frac{\pi}{4}\right) \cos x + \sin\left(\frac{\pi}{4}\right) \sin x = 1$ $\cos\left(\frac{\pi}{4} - x\right) = 1$ <p>لدينا: $r = \sqrt{2}$ منه:</p> $x - \frac{\pi}{4} = 2k\pi / k \in Z$ $x = \frac{\pi}{4} + 2k\pi / k \in Z$ <p>بالتالي: $S = \left\{\frac{\pi}{4} + 2k\pi / k \in Z\right\}$</p>
<p>$\cos a = \cos b \Leftrightarrow a = b + 2k\pi \text{ ou } a = -b + 2k\pi / k \in Z$</p> <p>$\cos a = 1 \Leftrightarrow a = 2k\pi / k \in Z$ $\cos a = -1 \Leftrightarrow a = \pi + 2k\pi / k \in Z$</p> <p>$\cos a = 0 \Leftrightarrow a = \frac{\pi}{2} + k\pi / k \in Z$</p> <p>$\sin a = \sin b \Leftrightarrow a = b + 2k\pi / k \in Z \text{ ou } a = \pi - b + 2k\pi / k \in Z$</p> <p>$\sin a = 1 \Leftrightarrow a = \frac{\pi}{2} + 2k\pi / k \in Z$ $\sin a = -1 \Leftrightarrow a = \frac{-\pi}{2} + 2k\pi / k \in Z$</p> <p>$\sin a = 0 \Leftrightarrow a = k\pi / k \in Z$</p>	

نذكر بالقواعد:

$\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 0$ $\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = 0$ <p>بالتالي: $S = \left\{\frac{\pi}{4} + k\pi / k \in Z\right\}$</p> $\cos\left(\frac{\pi}{4} + x\right) = 0$ $\frac{\pi}{4} + x = \frac{\pi}{2} + k\pi / k \in Z$ $x = \frac{\pi}{4} + k\pi / k \in Z$	<p>لنحل المعادلة: $\sin x - \cos x = 0$ ، لدينا:</p>
<p>لدينا: $f(x) = \frac{\cos x + \sin x}{\cos x - \sin x}$</p> <p>إذن: $Df = \{x \in IR / \cos x - \sin x \neq 0\} = IR - \left\{\frac{\pi}{4} + k\pi / k \in Z\right\}$</p>	<p>2</p>
$f(x) = \frac{(\cos x + \sin x)}{(\cos x - \sin x)} = \frac{(\cos x + \sin x)(\cos x + \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x + \sin^2 x + 2 \sin x \cos x}{\cos x \cos x - \sin x \sin x} = \frac{1 + \sin 2x}{\cos(x+x)} = \frac{1 + \sin 2x}{\cos 2x}$	<p>لدينا: 3</p>

لنحل المعادلة: $f(x) - \sqrt{3} = 0$ ، لدينا :

$$f(x) = \sqrt{3} \Leftrightarrow \frac{1 + \sin 2x}{\cos 2x} = \sqrt{3} \Leftrightarrow 1 + \sin 2x = \sqrt{3} \cos 2x \Leftrightarrow \sqrt{3} \cos 2x - \sin 2x = 1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x = \frac{1}{2} \Leftrightarrow \cos\left(\frac{\pi}{6}\right) \cos 2x - \sin\left(\frac{\pi}{6}\right) \sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6} + 2x\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{\pi}{6} + 2x = \frac{\pi}{3} + 2k\pi / k \in Z \text{ ou } \frac{\pi}{6} + 2x = \frac{-\pi}{3} + 2k\pi / k \in Z$$

$$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi / k \in Z \text{ ou } 2x = \frac{-\pi}{6} + 2k\pi / k \in Z$$

$$\Leftrightarrow x = \frac{\pi}{12} + k\pi / k \in Z \text{ ou } x = \frac{-\pi}{12} + k\pi / k \in Z$$

$$S = \left\{ \frac{\pi}{12} + k\pi / k \in Z \right\} \cup \left\{ \frac{-\pi}{12} + k\pi / k \in Z \right\} \text{ بالنالي:}$$

4

$$A(x) = \sqrt{3} \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right) = 2 \left(\frac{\sqrt{3}}{2} \cos\left(2x - \frac{\pi}{3}\right) - \frac{1}{2} \sin\left(2x - \frac{\pi}{3}\right) \right)$$

$$= 2 \left(\cos\left(\frac{\pi}{6}\right) \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right) \sin\left(2x - \frac{\pi}{3}\right) \right)$$

$$= 2 \cos\left(\frac{\pi}{6} + 2x - \frac{\pi}{3}\right) = 2 \cos\left(2x - \frac{\pi}{6}\right)$$

1

$$A(x) = 1 \Leftrightarrow 2 \cos\left(2x - \frac{\pi}{6}\right) = 1 \Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow 2x - \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi / k \in Z \text{ ou } 2x - \frac{\pi}{6} = \frac{-\pi}{3} + 2k\pi / k \in Z$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi / k \in Z \text{ ou } 2x = \frac{-\pi}{6} + 2k\pi / k \in Z$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi / k \in Z \text{ ou } x = \frac{-\pi}{12} + k\pi / k \in Z$$

2

لتحديد الحلول الموجودة في المجال $]-\pi; \pi]$ نؤطر العدد النسبي k ونحدد قيمته:

$$\frac{\pi}{4} + k\pi \in]-\pi; \pi] \Leftrightarrow -\pi < \frac{\pi}{4} + k\pi \leq \pi \Leftrightarrow -4\pi < \pi + 4k\pi \leq 4\pi \Leftrightarrow -5\pi < 4k\pi \leq 3\pi$$

$$\Leftrightarrow -5 < 4k \leq 3 \Leftrightarrow \frac{-5}{4} < k \leq \frac{3}{4} \Leftrightarrow k = -1 \text{ ou } k = 0$$

$$\Leftrightarrow x = \frac{\pi}{4} - \pi = \frac{-3\pi}{4} \text{ ou } x = \frac{\pi}{4}$$

وأيضا:

تمرين 12

و أيضا:

$$\frac{-\pi}{12} + k\pi \in]-\pi; \pi] \Leftrightarrow -\pi < \frac{-\pi}{12} + k\pi \leq \pi \Leftrightarrow -12\pi < -\pi + 12k\pi \leq 12\pi \Leftrightarrow -11\pi < 12k\pi \leq 13\pi$$

$$\Leftrightarrow -11 < 12k \leq 13 \quad \Leftrightarrow \frac{-11}{12} < k \leq \frac{13}{12} \Leftrightarrow k=0 \text{ ou } k=1$$

$$\Leftrightarrow x = \frac{-\pi}{12} \text{ ou } x = \frac{-\pi}{12} + \pi = \frac{11\pi}{12}$$

$$S = \left\{ \frac{\pi}{4}; \frac{-3\pi}{4}; \frac{-\pi}{12}; \frac{11\pi}{12} \right\} \text{ بالتالي:}$$

2

تمرين 13

$$A(x) = -2 \cos^2(x) + \sqrt{3} \sin(2x) + 2 = 2(1 - \cos^2 x) + \sqrt{3} \times 2 \sin x \cos x$$

$$= 2 \sin^2 x + 2\sqrt{3} \sin x \cos x = 4 \sin x \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right) \quad \text{لدينا:}$$

$$= 4 \sin(x) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right)$$

1

$$A(x) = 0 \Leftrightarrow 4 \sin(x) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right) = 0$$

$$\Leftrightarrow \sin(x) = 0 \text{ ou } \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right) = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \text{ ou } \cos\left(\frac{\pi}{6}\right) \cos x + \sin\left(\frac{\pi}{6}\right) \sin x = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \text{ ou } \cos\left(x - \frac{\pi}{6}\right) = 0$$

$$\Leftrightarrow x = k\pi / k \in Z \text{ ou } x - \frac{\pi}{6} = \frac{\pi}{2} + k\pi / k \in Z$$

$$\Leftrightarrow x = k\pi / k \in Z \text{ ou } x = \frac{2\pi}{3} + k\pi / k \in Z$$

$$S = \{k\pi / k \in Z\} \cup \left\{ \frac{2\pi}{3} + k\pi / k \in Z \right\} \text{ بالتالي:}$$

2