

:-_____-(3)

: E B A :_____ •

$$A \subset B \Leftrightarrow (\forall x \in E : x \in A \Rightarrow x \in B) \Leftrightarrow \forall x \in A : x \in B$$

$$. A \not\subset B \Leftrightarrow \exists x \in E / x \in A \quad x \notin B :$$

:_____ •

$$A = B \Leftrightarrow (A \subset B \quad B \subset A) \Leftrightarrow \forall x \in E : (x \in A \Leftrightarrow x \in B) :$$

$$. A \neq B \Leftrightarrow (A \not\subset B \quad B \not\subset A) :$$

:_____ •

$$. B \not\subset A \quad A \subset B : \quad A \subsetneq B \quad A \quad B$$

:03_____ •

$$B = \left\{ \frac{-1+2k}{3} / k \in \mathbb{Z} \right\} \quad A = \{1+2k / k \in \mathbb{Z}\} :$$

$$. A \subsetneq B :$$

:_____-(4)

$$. E \quad A \quad \{x \in E / x \notin A\}$$

$$. \forall x \in E : x \in \bar{A} \Leftrightarrow x \notin A : \quad C_E^A \quad \bar{A}$$

:_____ •

$$I \quad P$$

$$. (\bar{I} = P) \quad C_N^I = P \quad (\bar{P} = I) \quad C_N^P = I :$$

:_____ •

$$: E \quad B \quad A$$

$$A \subset B \Leftrightarrow (\forall x \in E : x \in A \Rightarrow x \in B) \Leftrightarrow (\forall x \in E : x \notin B \Rightarrow x \notin A)$$

$$. A \subset B \Leftrightarrow (\forall x \in E : x \in \bar{B} \Rightarrow x \in \bar{A}) \Leftrightarrow \bar{B} \subset \bar{A} :$$

$$. A \subset B \Leftrightarrow \bar{B} \subset \bar{A} :$$

:_____ •

$$. E \quad B \quad A$$

$$A \cap B \quad B \quad A \quad \{x \in E / x \in A \text{ و } x \in B\}$$

-I المجموعات:

:-_____-(1)

:_____ •

24

D_{24}

$$. D_{24} = \{d \in \mathbb{N} / \exists k \in \mathbb{N}, 24 = k d\} :$$

$$. D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\} :$$

$$P = \{n \in \mathbb{N} / \exists k \in \mathbb{N}, n = 2k\} :$$

P

$$. P = \{2k / k \in \mathbb{N}\} :$$

P

:_____ •

$$I = \{x \in \mathbb{R} / 0 < x \leq 1\} :$$

I

$]0,1]$ I

:01_____ •

-أ

$$E = \{1, 3, 5, 7, \dots\} : \quad F \quad E$$

-أ

$$. F = \{1, 10, 100, 1000, \dots\}$$

ID

Q

-ب

:_____-(2)

:_____ •

$$E = \{-2, -1, 1, 3, 5\} \quad B = \{-1, 0, 1\} \quad A = \{-2, 1, 5\} :$$

$$E \quad (\quad) \quad A \quad E \quad A$$

$$. (A \quad E) \quad E \quad A \quad A \subset E$$

$$. B \not\subset E : \text{نكتب } B \quad E \quad E \text{ ينتمي إلى } B \text{ و لا ينتمي إلى } E$$

:_____ •

E

E

$$. P(E) = \{A / A \subset E\} : \quad P(E) \quad E$$

$$: \quad E \quad E = \{a, b, c\} :_____ •$$

$$. P(E) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, E\}$$

:02_____ •

$$. E = \{a, b, 1, 2\}$$

:02 •

$$A \setminus B = A \cap \bar{B} \quad A \setminus \bar{B} = A \cap B \quad A \Delta E = \bar{A} \quad \bar{\bar{A}} = A$$

$$A \cup B = E \Leftrightarrow \bar{A} \subset B \Leftrightarrow \bar{B} \subset A \quad A \cap B = \emptyset \Leftrightarrow A \subset \bar{B} \Leftrightarrow B \subset \bar{A}$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C) \quad A \Delta (B \Delta C) = (A \Delta B) \Delta C$$

:06 •

$$A \setminus B = A \Leftrightarrow B \setminus A = B \quad A = B \Leftrightarrow A \cup B = A \cap B \quad \bar{A} \setminus \bar{B} = B \setminus A$$

$$A \cup B = A \cap C \Leftrightarrow B \subset A \subset C \quad A \Delta B = A \Delta C \Leftrightarrow B = C$$

$$(A \cup B = A \cup C \text{ و } A \cap B = A \cap C) \Leftrightarrow B = C$$

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C) \quad A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

:07 •

$$B \subsetneq A \subsetneq C \quad E \quad C \quad B \quad A \quad \text{أ-}$$

$$(1) : \begin{cases} A \cap X = B \\ A \cup X = C \end{cases} : P(E)$$

$$(2) : \begin{cases} A \setminus X = B \\ X \setminus A = C \end{cases} : \text{ب-}$$

$$B \subsetneq A \subsetneq \bar{C}$$

-II الجداء الديكارتي لمجموعتين:

$$(x, y) \quad F \quad E$$

$$E \times F \quad F \quad E \quad y \in F \quad x \in E$$

$$E \times F = \{(x, y) / x \in E \text{ و } y \in F\} :$$

$$E^2 \quad E \times E \quad F = E$$

$$E^2 = \{(x, y) / x \in E; y \in E\}$$

:01 •

$$E = \{1, 2\} \quad F^2 \quad E^2 \quad F \times E \quad E \times F :$$

$$\forall x \in E : x \in A \cap B \Leftrightarrow (x \in A \text{ و } x \in B) :$$

$$A \cap B = \emptyset : \quad B \quad A$$

$$A \cup B = \{x \in E / x \in A \text{ و } x \in B\} \quad B \quad A$$

$$\forall x \in E : x \in A \cup B \Leftrightarrow (x \in A \text{ أو } x \in B) :$$

:04 •

$$A \cup B = A \Leftrightarrow B \subset A \quad A \cap B = A \Leftrightarrow A \subset B :$$

:01 •

$$\forall x \in E : x \notin A \cap B \Leftrightarrow (x \notin A \text{ و } x \notin B) \Leftrightarrow (x \in \bar{A} \text{ و } x \in \bar{B}) :$$

$$\forall x \in E : x \in \overline{A \cap B} \Leftrightarrow x \in \bar{A} \cup \bar{B} :$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B} :$$

$$\overline{A \cup B} = \bar{A} \cap \bar{B} :$$

:01 (قانون موركان) •

$$\overline{A \cap B} = \bar{A} \cup \bar{B} : \quad E \quad B \quad A$$

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

:01 •

$$A \setminus B = \{x \in E / x \in A \text{ و } x \notin B\} : \quad B \quad A \quad E$$

$$B \setminus A = \{x \in E / x \in B \text{ و } x \notin A\}$$

$$(A \setminus B) \cap (B \setminus A) = \emptyset : \quad B \setminus A \quad A \setminus B$$

$$A \Delta B = (A \setminus B) \cup (B \setminus A) :$$

$$A \Delta B = (A \setminus B) \cup (B \setminus A) :$$

$$A \Delta B = (A \cup B) \setminus (A \cap B) : \quad B \Delta A = A \Delta B$$

:05 •

$$B = \left\{ \frac{5+8k}{20} / k \in \mathbb{Z} \right\} \quad A = \left\{ \frac{5+4k}{10} / k \in \mathbb{Z} \right\} : \quad \text{أ-}$$

$$(A \cap B = \emptyset)$$

$$A \cup B = \{1, 2, 3, \dots, 10, 11\} : \quad B \quad A \quad \text{ب-}$$

$$A \setminus B = \{7, 8, 9, 10\} \quad A \cap B = \{4, 5, 6, 11\}$$

• $E \quad x \quad id_E$

• $\forall x \in E : f(x) = \alpha : F \quad \alpha \quad f : E \rightarrow F$

• $E \quad f = g \quad g \quad f$

• $\forall x \in E : f(x) = g(x) :$

• $F \quad E \quad g \quad E \quad E' \quad f$

• $\forall x \in E' : g(x) = f(x) :$

• $F \quad E'' \quad h \quad E \quad E'' \quad f$

• $\forall x \in E : h(x) = f(x) :$

• $x \quad E(x) \quad f : \begin{cases} \mathbb{R} \rightarrow \mathbb{Z} \\ x \mapsto E(x) \end{cases} :$

• $\mathbb{Z} \quad id_{\mathbb{Z}} \quad \mathbb{Z} \quad f \quad \forall k \in \mathbb{Z} : E(k) = k :$

• $I_k \quad f \quad f_k \quad I_k = [k, k+1[: \mathbb{Z} \quad k$

• $\forall x \in I_k : f_k(x) = k : \quad \forall x \in I_k : E(x) = k :$

• $k \quad I_k \quad f$

• $F \quad E \quad B \quad A \quad f : E \rightarrow F$

• $A \quad F \quad \{f(x) / x \in A\}$

• $f(A)$

• $B \quad E \quad \{x \in E / f(x) \in B\}$

• $f^{-1}(B)$

• $\forall y \in F : y \in f(A) \Leftrightarrow \exists x \in A / f(x) = y$

• $\forall x \in E : x \in f^{-1}(B) \Leftrightarrow f(x) \in B$

• $F = \{a, b, c\}$

• $E \times F = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\} :$

• $F \times E = \{(a,1), (b,1), (c,1), (a,2), (b,2), (c,2)\}$

• $E^2 = \{(1,1), (1,2), (2,1), (2,2)\}$

• $F^2 = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$

• $F \quad E \quad E \times F \neq F \times E \quad E \neq F$

• (O, \vec{i}, \vec{j})

• $J \times I \quad I \times J \quad J^2 \quad I^2$

• (3): $\begin{cases} I =]-\infty, -1] \\ J = [-1, +\infty[\end{cases} \quad (2): \begin{cases} I = \mathbb{R}_+ \\ J = \mathbb{R}_- \end{cases} \quad (1): \begin{cases} I = [-2, 3] \\ J = [-1, 2] \end{cases}$

• $F = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 1\} \quad E = [-1, 1] :$

• $F \subsetneq E^2 \quad F \neq \emptyset$

• $E = \{(x, y) \in \mathbb{R}^2 / x^2 \leq y \text{ و } y^2 \leq x\} :$ نعتبر المجموعة

• $E \subsetneq [0, 1] \times [0, 1]$ ، ثم بين أن $E \neq \emptyset$

• **III- التطبيقات**

• $- (1)$

• $F \quad E \quad f \quad F \quad E$

• $y = f(x) : (y \quad x) f \quad x \quad y$

• $f : \begin{cases} E \rightarrow F \\ x \mapsto y = f(x) \end{cases}$

$$\forall x \in E : h(x) = g[f(x)]$$

$$g \circ f : \begin{cases} E \rightarrow G \\ x \mapsto g \circ f(x) = g[f(x)] \end{cases} : \quad g \circ f \quad h$$

:_____ •
:

$$g : \begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x^2 + 2x \end{cases} \quad f : \begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x^2 - 2x \end{cases}$$

$$f \circ g \quad g \circ f$$

$$f \circ f : \begin{cases} E \rightarrow E \\ x \mapsto f[f(x)] \end{cases} : \quad E \quad E \quad f \quad -$$

$$f^4 = f^3 \circ f \quad f^3 = f^2 \circ f : \quad f^2 \quad f \circ f$$

$$: \quad h : G \rightarrow H \quad g : F \rightarrow G \quad f : E \rightarrow F \quad -$$

$$. (\quad) \quad h \circ (g \circ f) = (h \circ g) \circ f$$

:_____ - (3)

:_____ •

() $f : E \rightarrow F$ إذا كان المجموعة E لا تتضمن عنصريين

مختلفين لهما نفس الصورة ب f ، بعبارة أخرى :

$$\forall (x, x') \in E^2 : f(x) = f(x') \Rightarrow x = x'$$

$$\forall (x, x') \in E^2 : x \neq x' \Rightarrow f(x) \neq f(x') \quad \text{أو}$$

:04_____ •

$$f^{-1}(\{y\}) \quad f : E \rightarrow F$$

. $F \quad y$

:11_____ •

$$g : \begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{x}{1+|x|} \end{cases} \quad f : \begin{cases} \mathbb{N} \rightarrow \mathbb{N} \\ n \mapsto n + (-1)^n \end{cases} :$$

$$h : \begin{cases} \mathbb{R} \rightarrow \mathbb{Z} \\ x \mapsto E(x) \end{cases} :$$

$$. x \in f^{-1}(\{y\}) \Leftrightarrow f(x) = y : \quad E \times F \quad (x, y)$$

$$. y \quad f^{-1}(\{y\}) :$$

:01_____ •

$$. f : \begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x^2 \end{cases} :$$

$$. f(]-\infty, -10[) \quad f(\mathbb{Z}) \quad f([-3, 2])$$

:02_____ •

$$. f : \begin{cases} \mathbb{R} \rightarrow \mathbb{Z} \\ x \mapsto E(x) \end{cases} :$$

: $\mathbb{Z} \quad k$

$$x \in f^{-1}(\{k\}) \Leftrightarrow f(x) = k \Leftrightarrow E(x) = k \Leftrightarrow k \leq x < k + 1$$

$$. f^{-1}(\{k\}) = [k, k + 1[:$$

:03_____ •

$$. F \quad E \quad f$$

$$: \quad F \quad E \quad B \quad A \quad -$$

$$. f(f^{-1}(B)) \subset B \quad A \subset f^{-1}(f(A))$$

: $E \quad A' \quad A \quad -$

$$. f(A \cap A') \subset f(A) \cap f(A') \quad f(A \cup A') = f(A) \cup f(A')$$

: $F \quad B' \quad B \quad -$

$$. f^{-1}(B \cap B') = f^{-1}(B) \cap f^{-1}(B') \quad f^{-1}(B \cup B') = f^{-1}(B) \cup f^{-1}(B')$$

:10_____ •

$$. f : E \rightarrow F$$

$$. f^{-1}(\overline{B}) = \overline{f^{-1}(B)} : \quad F \quad B$$

:_____ - (2)

:_____ •

$$. g : F \rightarrow G \quad f : E \rightarrow F :$$

$$: \quad G \quad E \quad h \quad g \quad f$$

$$g(n) = \frac{n-1}{2} \quad n \quad g(n) = \frac{n}{2}$$

$$f : E \rightarrow F$$

$$\forall y \in F, \exists! x \in E / y = f(x)$$

$$f : \mathbb{N} \rightarrow \mathbb{N}$$

$$n \mapsto n + (-1)^n$$

$$g : \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto \frac{x}{1+|x|}$$

$$h : \mathbb{R} \rightarrow \mathbb{Z}$$

$$x \mapsto E(x)$$

$$f(\overline{A}) = \overline{f(A)} : E \rightarrow A$$

$$f^{-1} : F \rightarrow E$$

$$y \mapsto x / f(x) = y$$

$$f \circ f^{-1} = id_F \quad f^{-1} \circ f = id_E$$

$$g : F \rightarrow G \quad f : E \rightarrow F$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

و إذا كان $g \circ f$

$$f(A \cap B) = f(A) \cap f(B)$$

$$A \cap B = \emptyset \Rightarrow f(A) \cap f(B) = \emptyset$$

$$f(\overline{A}) \subset \overline{f(A)}$$

$$f^{-1}(f(A)) = A$$

$$(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv) \Rightarrow (v) \Rightarrow (i)$$

$$f^{-1}(\{y\}) \neq \emptyset$$

$$f(E) = F$$

$$h : \mathbb{R} \rightarrow \mathbb{Z}$$

$$x \mapsto E(x)$$

$$f : \mathbb{N} \rightarrow \mathbb{N}$$

$$n \mapsto n + (-1)^n$$

$$g : \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto \frac{x}{1+|x|}$$

$$f(\overline{A}) \subset \overline{f(A)}$$

$$f(f^{-1}(B)) = B$$

$$f \circ g \quad g \circ f \quad g \circ f$$

$$f(n) = 2n$$

• :14

$$f : \mathbb{Z} \rightarrow \mathbb{N} \quad f(n) = -\frac{n+1}{2}$$

$$f^{-1} : \mathbb{N} \rightarrow \mathbb{Z} \quad f^{-1}(n) = \frac{n}{2}$$

• :15

$$f : \mathcal{P}(E) \rightarrow \mathcal{P}(A) \times \mathcal{P}(B)$$

$$f : X \mapsto (X \cap A, X \cap B)$$

(f $\Leftrightarrow A \cap B = \emptyset$) (f $\Leftrightarrow A \cup B = E$) :

• :16

$$f \circ f \circ f = f : E \rightarrow E$$

(f) \Leftrightarrow (f) :

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ثانوية الفتح - الخميسات