

:03

(3): $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin 3x}{1 - 2 \cos x}$ (2): $\lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1+x})}{\sin^2 2x}$ (1): $\lim_{x \rightarrow 0} \frac{3x^2 - x^3}{1 - \cos 3x}$

(6): $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\tan 3x}{1 - 2 \cos x}$ (5): $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \cos x}$ (4): $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \sqrt{2} \cos x}{1 + \sqrt{2} \sin x}$

(8): $\lim_{x \rightarrow \pi} \frac{1 - \cos x \cos 3x}{1 + \cos^3 x}$ (7): $\lim_{x \rightarrow \frac{1}{2}} \left(x^2 - \frac{1}{4} \right) \tan(\pi x)$

:04

$f(x) = \frac{\sin 2x}{x^3 + |x|}$: f - (1)

$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = -1$
 $x_0 = 0$ f - ب

x_0 g f - (2)

(2): $f(x) = \frac{2 - \sqrt{-2x}}{2 - \sqrt{x+6}}$; $x_0 = -2$ (1): $f(x) = \frac{1 - \sqrt{-x}}{1 + x^3}$; $x_0 = -1$

(3): $f(x) = \frac{\sqrt{2x+5} - \sqrt{3x+3}}{(4x+1)(x-2)}$; $x_0 = 2$

(2) - ملخص

:

I x_0 I f

$\lim_{x \rightarrow x_0} f(x) = f(x_0)$: x_0 f

I I f

:01

\mathbb{R} -
 $-\infty$ $+\infty$ -
 $-\infty$ $+\infty$ -

I - تذكير

(1) - أنشطة

:01

(1) D_f - (1) عند x_0 في كل حالة من الحالات التالية :

(2): $f(x) = \frac{-2x^2 - x + 6}{(x+2)^3}$ و $x_0 = -2$ (1): $f(x) = \frac{x^2 + x - 6}{-3x^2 - 7x + 6}$ و $x_0 \in \left\{ -3, \frac{2}{3} \right\}$

(3): $f(x) = \frac{x^2 - 2|x|}{x^3 + |x|}$ و $x_0 \in \{-1, 0\}$

(2) - حدد تبعا لقيم العدد n من \mathbb{N} نهاية الدالة $g : x \mapsto x^n - x^{11} - 3x^8 + 10$ عند $-\infty$

(3) - حدد تبعا لقيم العدد n من \mathbb{N} نهاية الدالة $h : x \mapsto \frac{x^n - x^8 - x^4 + 3}{x^8 + 4}$ عند $-\infty$

:02

(1) :

(3): $\lim_{x \rightarrow 9} \frac{x\sqrt{x} - 27}{3 - \sqrt{x}}$ (2): $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{x + \sqrt{x} - 6}$ (1): $\lim_{x \rightarrow 3} \frac{2x - 6}{2 - \sqrt{x+1}}$

(5): $\lim_{x \rightarrow \pm\infty} \frac{2|x| - \sqrt{4x^2 - 1}}{x+1}$ (4): $\lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{-2x^2 + x + 6}$

(7): $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 1} - \sqrt{x^2 + 2}}{\sqrt{x^2 - 2} - \sqrt{x^2 + 1}}$ (6): $\lim_{x \rightarrow +\infty} \sqrt{x + \sqrt{x}} - \sqrt{x - 2\sqrt{x}}$

(2) - نعتبر الدالة العددية f :

$f(x) = \frac{2 - \sqrt{3+x^2}}{x - \sqrt{x}}$

$\lim_{x \rightarrow +\infty} f(x)$ $\lim_{x \rightarrow 0^+} f(x)$: D_f - أ

g $x_0 = 1$ f - ب

ب- $\forall x \in]-\infty, 0[: f(x) \leq \frac{x\sqrt{3-x}}{3}$:

ج- $\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} f(x)$:

(3) $\lim_{x \rightarrow -\infty} \frac{x^3}{3x - \cos x}$ (2) $\lim_{x \rightarrow +\infty} \frac{2x + \sin x}{x - 1}$ (1) $\lim_{x \rightarrow +\infty} \frac{2x^2 - x \cos x + 1}{x^2 + 2}$

(4) $\lim_{x \rightarrow -\infty} \frac{1 - x^3}{1 + x^2 \sin^2 x}$

II - نهاية مركب دالتين عدديتين:

$f(I) \subseteq J$ $J \ I$ $g \ f$

الدالة h المعرفة على I : $h(x) = g[f(x)]$

$g \circ f : I \rightarrow \mathbb{R}$ $x \mapsto g \circ f(x) = g[f(x)]$: h ب $g \circ f$ ولدينا

و بصفة عامة ، إذا كانت f g :

$\forall x \in D_{g \circ f} : g \circ f(x) = g[f(x)]$ $D_{g \circ f} = \{x \in \mathbb{R} / x \in D_f \text{ و } f(x) \in D_g\}$

(1) - نهاية مركب دالتين عدديتين:

:06

$g \ x_0 \ I$ f

$\lim_{x \rightarrow x_0} f(x) = y_0$ و $\lim_{y \rightarrow y_0} g(y) = L$ $\Rightarrow \lim_{x \rightarrow x_0} g \circ f(x) = L$

حيث $y_0 \in \mathbb{R} \cup \{-\infty, +\infty\}$ L

$x_0 \ I$

:02

$f : x \mapsto \tan x \quad \mathbb{R} \quad v : x \mapsto \cos x \quad u : x \mapsto \sin x$

$D_f = \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi / k \in \mathbb{Z} \right\}$

نهايات مثلثية مرجعية:

$\lim_{x \rightarrow 0} \frac{1 - \cos(ax)}{x^2} = \frac{a^2}{2}$ $\lim_{x \rightarrow 0} \frac{\tan(ax)}{bx} = \frac{a}{b}$ $\lim_{x \rightarrow 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$

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$x_0 \ I$ f

$\lim_{x \rightarrow x_0} \sqrt{f(x)} = \sqrt{L}$ $L \geq 0$ $I \ f \geq 0$ $\lim_{x \rightarrow x_0} f(x) = L$

:04

$|f| : x \mapsto |f(x)|$ I f

$\sqrt{f} : x \mapsto \sqrt{f(x)}$ I f

(3) - النهايات و الترتيب:

:05

$x_0 \ I$ و ثلاث دوال

عددية $f \ g \ h$: $\forall x \in I : g(x) \leq f(x) \leq h(x)$:

$\lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} h(x) = L \Rightarrow \lim_{x \rightarrow x_0} f(x) = L$

$\lim_{x \rightarrow x_0} h(x) = -\infty \Rightarrow \lim_{x \rightarrow x_0} f(x) = -\infty$ $\lim_{x \rightarrow x_0} g(x) = +\infty \Rightarrow \lim_{x \rightarrow x_0} f(x) = +\infty$

:05

$f(x) = \frac{x\sqrt{3-x}}{2 + \sin\left(\frac{1}{x}\right)}$ f

$\lim_{x \rightarrow 0} f(x) \quad \forall x \in]-1, 1[- \{0\} : |f(x)| < 2|x| : D_f$ أ-

• :_____

• $h(x) = \sin\left(\frac{x^3}{x^2 - 4x + 3}\right) : h$

$g(x) = \sin x \quad f(x) = \frac{x^3}{x^2 - 4x + 3} \quad h = g \circ f :$

g و الدالة () $D_f = \mathbb{R} - \{1,3\}$ f
 $D_f = \mathbb{R} - \{1,3\}$ متصلة على \mathbb{R} وبما أن $f(D_f) \subseteq \mathbb{R}$ فإن $h = g \circ f$
 $D_h = \mathbb{R} - \{1,3\}$ h

• :07_____

• $D_h \quad h(x) = \tan\left(\frac{\pi}{2x}\right) : h$

(3) - نهاية مركب دالة متصلة و دالة تقبل نهاية:

• :08_____

$g \quad x_0 \quad I \quad f$
 $: \quad f(I) \subseteq J \quad J$

$(L \quad g \quad \lim_{x \rightarrow x_0} f(x) = L) \Rightarrow \lim_{x \rightarrow x_0} g \circ f(x) = g(L)$

• :_____

$x_0 \quad I$

• :08_____

(2): $\lim_{x \rightarrow -1} \sqrt{\frac{-5x^2 + 2x + 7}{x^3 + 1}}$ (1): $\lim_{x \rightarrow 0} \tan\left(\frac{\sin \pi x}{3x}\right)$

(4): $\lim_{x \rightarrow -2} \sin\left(\frac{\pi(x^2 - 4)}{x^3 + 8}\right)$ (3): $\lim_{x \rightarrow -\infty} \cos\left(\frac{\pi x^2 - 2x + 3}{3x^2 + 2}\right)$

• (5): $\lim_{x \rightarrow 0} \cos\left(\frac{\pi x}{2(1 - \sqrt{1+x})}\right)$

• :01_____

• $\lim_{x \rightarrow +\infty} x \left(1 - \cos \frac{1}{\sqrt{x}}\right)$

$g(x) = \frac{1 - \cos x}{x^2} \quad f(x) = \frac{1}{\sqrt{x}} : g \quad f$

$\begin{cases} \lim_{x \rightarrow +\infty} f(x) = 0 \\ \lim_{x \rightarrow 0} g(x) = \frac{1}{2} \end{cases} \quad \lim_{x \rightarrow +\infty} x \left(1 - \cos \frac{1}{\sqrt{x}}\right) = \lim_{x \rightarrow +\infty} g \circ f(x) :$

$\lim_{x \rightarrow +\infty} x \left(1 - \cos \frac{1}{\sqrt{x}}\right) = \frac{1}{2} :$

• :02_____

$f(x) = \frac{x+1}{2x-1} : \mathbb{R} - \left\{\frac{1}{2}\right\}$

• $\lim_{x \rightarrow -\infty} f \circ f(x) \quad \lim_{x \rightarrow +\infty} f \circ f(x) :$

$\left(\left[\frac{1}{2}, +\infty\right[\quad f(x) > \frac{1}{2}\right) \text{ مع } \lim_{x \rightarrow +\infty} f(x) = \frac{1}{2} :$

• $\lim_{x \rightarrow +\infty} f \circ f(x) = +\infty$: إذن $\lim_{x \rightarrow \left(\frac{1}{2}\right)^+} f(x) = +\infty :$

• :06_____

$\lim_{x \rightarrow -\infty} (x - \sqrt{x^2 - 1}) \cos(x + \sqrt{x^2 - 1}) :$

• $\lim_{x \rightarrow -\infty} x \cos(x + \sqrt{x^2 - 1})$

(2) - إتصال مركب دالتين عدديتين:

• :07_____

$f(I) \subseteq J \quad J \quad I \quad g \quad f$
 $x_0 \quad g \circ f \quad f(x_0) \quad g \quad x_0 \quad f$
 $I \quad g \circ f \quad J \quad g \quad I \quad f$

• :_____

• : [a,b] f

• [a,b] f f([a,b]) = [f(a), f(b)]

• [a,b] f f([a,b]) = [f(b), f(a)]

• (3) - ميرهنة القيم الوسيطية: :01

• f(b) f(a) k [a,b] f

• [a,b] f(x) = k

• :_____

• :11

• f(x) = k [a,b] f

• f(b) f(a) k [a,b]

• :_____

• f

• :_____

• [lim_{x→a^-} f(x), lim_{x→-∞} f(x)] k]-∞, a[f

•]-∞, a[f(x) = k

• :12

• f(a) × f(b) < 0 [a,b] f

• [a,b] f(x) = 0

• :_____

•]0,1[(E) : x^5 + x^3 + 3x - 4 = 0 :

• f(x) = x^5 + x^3 + 3x - 4 :

• f(0)f(1) < 0 : f(1) = 1 f(0) = -4 :

• [0,1] ℝ f

• :_____ -III

• (1) - أنشطة: :09

• f(x) = { 1-x^2; x ≤ 1 ; ℝ f

• { x-1; x > 1

• (O, i, j) (C_f) ℝ f أ

• J = [-3, -1] I = [1, 4] : f ب

• N =]-∞, 4[M =]-∞, 0[L = [2, +∞[K = [-2, 1]

• :10

• g(x) = (x+1)/(x-1) : ℝ - {1} g

• g(]-∞, -1]) g]3, 5[g(]-∞, 1[) g ([1, 5])

• (2) - ملخص: :09

• ℝ

• :_____

• f(x) = { -1; x ≤ 0 ; ℝ f

• { 1; x > 0

• I ∩]-∞, 0[≠ ∅ I ∩]0, +∞[≠ ∅ ℝ I

• f(I) f(I) = {-1, 1}

• (0 f)

• :10

• f([a,b]) = [m, M] : فإن ، (a < b) [a,b] f

• [a,b] f و ، M = sup_{x∈[a,b]} f(x) و m = inf_{x∈[a,b]} f(x) : حيث

• ∃ y ∈ [m, M] ∃ x ∈ [a,b] / y = f(x) : [m, M]

• _____ :

$$f(x) = \frac{x^2}{x+1} : \mathbb{R} - \{-1\} \quad f$$

• $K =]-1, 0]$ f h $I =]-\infty, -2]$ f g

• J I g I g أ-

• L K h K h ب-

• h g h^{-1} g^{-1} ج-

(1) - دالة قوس الظل:

$$I = \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\quad f : x \mapsto \tan x$$

$$J = f \left(\left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\right) \quad I$$

$J = \mathbb{R} : \lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = +\infty \quad \lim_{x \rightarrow -\frac{\pi}{2}^+} \tan x = -\infty :$

$\mathbb{R} \quad \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\quad f : x \mapsto \tan x$

$: \quad \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\quad \mathbb{R} \quad f^{-1}$

$\forall x \in \mathbb{R} \quad \forall y \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[: f^{-1}(x) = y \Leftrightarrow \tan y = x$

• _____ :

$\mathbb{R} \quad \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\quad f : x \mapsto \tan x$

$\arctan : \left[\mathbb{R} \rightarrow \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\right. : \arctan$

$\left. \begin{matrix} x \mapsto y / \tan y = x \\ \forall x \in \mathbb{R} \quad \forall y \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[: y = \arctan x \Leftrightarrow \tan y = x \end{matrix} \right.$

• $]0, 1[\quad (E) \quad f(x) = 0$

• 13 : $f(a) \times f(b) < 0 \quad [a, b]$ f

• 11 : $[a, b] \quad f(x) = 0$

• $\left] \frac{1}{4}, \frac{1}{2} \right[\quad \alpha \quad (E) : x^3 + 2x - 1 = 0$

-IV الدوال العكسية لدوال متصلة و رتيبة قطعاً على مجال:

• 02 :

$J = f(I) \quad I \quad f \quad I \quad f$

$f \quad J \quad f^{-1} : \begin{matrix} J \rightarrow I \\ x \mapsto y / f(y) = x \end{matrix} :$

• $(O, \vec{i}, \vec{j}) \quad (C_f) \quad (C_{f^{-1}})$

• _____ :

$: f \quad I \quad J = f(I)$

$J = f(I)$		I
I	f	f
$\left] \lim_{x \rightarrow b^-} f(x), f(a) \right[$	$\left[f(a), \lim_{x \rightarrow b^-} f(x) \right[$	$[a, b[$
$\left] \lim_{x \rightarrow b^-} f(x), \lim_{x \rightarrow a^+} f(x) \right[$	$\left] \lim_{x \rightarrow a^+} f(x), \lim_{x \rightarrow b^-} f(x) \right[$	$]a, b[$
$\left] \lim_{x \rightarrow +\infty} f(x), f(a) \right[$	$\left[f(a), \lim_{x \rightarrow +\infty} f(x) \right[$	$[a, +\infty[$
$\left] \lim_{x \rightarrow b^-} f(x), \lim_{x \rightarrow -\infty} f(x) \right[$	$\left] \lim_{x \rightarrow -\infty} f(x), \lim_{x \rightarrow b^-} f(x) \right[$	$] -\infty, b[$
$\left] \lim_{x \rightarrow +\infty} f(x), \lim_{x \rightarrow -\infty} f(x) \right[$	$\left] \lim_{x \rightarrow -\infty} f(x), \lim_{x \rightarrow +\infty} f(x) \right[$	$\mathbb{R} =] -\infty, +\infty[$

ب- :

(3): $\lim_{x \rightarrow \pm\infty} \arctan\left(\frac{1-x^2}{1+\sqrt{3}x^2}\right)$ (2): $\lim_{x \rightarrow 0} \frac{\arctan(x^2-3x)}{x^3}$ (1): $\lim_{x \rightarrow 0} \frac{\arctan x}{x^2+2x}$

(4): $\lim_{x \rightarrow \pm\infty} x^3 \arctan\left(\frac{1}{x^2}\right)$

(2) - دالة الجذر من الرتبة n حيث n و n ≥ 2

$\mathbb{R} \quad \mathbb{N} \quad n$
 $x \mapsto x^n : n \geq 2$

$f(\mathbb{R}_+) = \mathbb{R}_+ : \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} x^n = +\infty \quad f(0) = 0$

$\mathbb{R}_+ \quad \mathbb{R}_+ \quad f : x \mapsto x^n \quad n \geq 2 \quad \mathbb{N} \quad n$

$\sqrt[n]{x} : [0, +\infty[\rightarrow [0, +\infty[$
 $x \mapsto y = \sqrt[n]{x} / y^n = x$

$\sqrt[3]{x} = \sqrt{x} : \mathbb{R}_+ \quad x$

$\lim_{x \rightarrow +\infty} \sqrt[n]{x} = +\infty : \mathbb{R}_+ \quad \sqrt[n]{x} : x \mapsto \sqrt[n]{x}$

$\sqrt{x} < \sqrt{y} \Leftrightarrow x < y \quad \sqrt{x} = \sqrt{y} \Leftrightarrow x = y \quad y = \sqrt{x} \Leftrightarrow y^n = x$

$(\sqrt[n]{x})^n = \sqrt[n]{x^n} = x : \mathbb{R}_+ \quad x$

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$(4): x^{10} = -5 \quad (3): x^5 = -32 \quad (2): x^4 = 32 \quad (1): x^7 = 128$

$a \in \mathbb{R} \quad (E): x^n = a : \mathbb{R}$

:14 •

$\mathbb{R} \quad \arctan$

$\forall x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[: \arctan(\tan x) = x \quad \forall x \in \mathbb{R} : \tan(\arctan x) = x$

$\forall (x, y) \in \mathbb{R}^2 : \begin{cases} \arctan x = \arctan y \Leftrightarrow x = y \\ \arctan x < \arctan y \Leftrightarrow x < y \end{cases}$

$\lim_{x \rightarrow 0} \frac{\arctan x}{x} = 1 \quad \lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2} \quad \lim_{x \rightarrow +\infty} \arctan x = \frac{\pi}{2}$

:12 •

$\arctan 2 + \arctan 5 + \arctan 8 = \frac{5\pi}{4} \quad \arctan\left(\frac{5}{2}\right) + \arctan\left(\frac{7}{3}\right) = \frac{3\pi}{4}$

:13 •

$\mathbb{R} \quad x$

$\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}} \quad \sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$

$2 \arctan(\sqrt{1+x^2} - x) + \arctan x = \frac{\pi}{2}$

$\mathbb{R}^* \quad x$

$sg(x) = \begin{cases} 1; x > 0 \\ -1; x < 0 \end{cases} : \arctan x + \arctan\left(\frac{1}{x}\right) = sg(x) \frac{\pi}{2}$

$\mathbb{R} \quad x$

$\cos^2\left(\frac{1}{2} \arctan x\right) = \frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}} \quad \cos(3 \arctan x) = \frac{1-3x^2}{(1+x^2)\sqrt{1+x^2}}$

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$\mathbb{R} \quad g \quad f$

$g(x) = \tan(3 \arctan x) \quad f(x) = \tan(2 \arctan x)$

$\forall x \in D_g : g(x) = \frac{x(3-x^2)}{1-3x^2} \quad \forall x \in D_f : f(x) = \frac{2x}{1-x^2}$

(3) - القوى الجذرية لعدد حقيقي موجب قطعاً:

• :_____

$$r = \frac{m}{n} \quad x$$

($n \in \mathbb{N}^*$ و $m \in \mathbb{Z}^*$)

العدد الحقيقي الموجب قطعاً $\sqrt[n]{x^m}$ يسمى القوة الجذرية للعدد x

• :_____

$x^r = x^{\frac{m}{n}} = \sqrt[n]{x^m}$: x^r

• :_____

$$\sqrt[n]{x} = \sqrt[n]{x^1} = x^{\frac{1}{n}} : \quad \mathbb{N}^* - \{1\} \quad n \quad \mathbb{R}_+^* \quad x$$

• :_____

$$\sqrt{x} = \sqrt[2]{x} = x^{\frac{1}{2}} :$$

• :_____

$$27^{\frac{4}{3}} = \frac{1}{81} \quad 5^{\frac{7}{6}} = 5\sqrt[6]{5} \quad 8^{\frac{2}{3}} = 4 :$$

• :_____

$$(x \cdot y)^r = x^r \cdot y^r \quad (x^r)^{r'} = x^{r \cdot r'} \quad x^{r+r'} = x^r \cdot x^{r'}$$

• :_____

$$\frac{x^r}{x^{r'}} = x^{r-r'} \quad x^{-r} = \frac{1}{x^r} \quad \left(\frac{x}{y}\right)^r = \frac{x^r}{y^r}$$

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• :_____

أ- $b = \sqrt[3]{3} \quad a = \sqrt[4]{2}$

• :_____

$$B = \frac{27^{\frac{2}{9}} \times 81^{\frac{1}{4}} \times 9^{\frac{5}{2}}}{3^{\frac{17}{3}}} \quad A = \frac{\sqrt[5]{162} \cdot \sqrt[4]{9} \cdot \sqrt{3}}{\sqrt[4]{128}}$$

• :_____

ب- $g : x \mapsto x^{\frac{2}{3}} \quad f : x \mapsto \sqrt[3]{x^2}$

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• العمليات على الجذور من الرتبة n :

• :_____

$$p \geq 2 \quad n \geq 2 \quad \mathbb{N}^{*2} \quad (n, p) \quad \mathbb{R}_+^2 \quad (x, y)$$

$$(\sqrt[n]{x})^p = \sqrt[n]{x^p} \quad \sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{x \cdot y} \quad \sqrt[n]{\sqrt[p]{x}} = \sqrt[n \cdot p]{x} \quad \sqrt[n]{x^p} = \sqrt[n]{x^p}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} / y \neq 0 \quad \sqrt[n]{x} \cdot \sqrt[p]{x} = \sqrt[n \cdot p]{x^{n+p}}$$

• :_____

$$n \geq 2 \quad \mathbb{N}^* \times \mathbb{Z}^* \quad (n, p) \quad \mathbb{R}_+^* \quad x \quad \sqrt[n \cdot p]{x^p} = \sqrt[n]{x^p} :$$

• :_____

إتصال مركب دالة عددية f و الدالة $\sqrt[n]{}$:

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• :_____

$$\sqrt[n]{f} : x \mapsto \sqrt[n]{f(x)} \quad I \quad f$$

• :_____

$$\lim_{x \rightarrow x_0} \sqrt[n]{f(x)} = \sqrt[n]{L} \text{ و } L \geq 0 \Rightarrow \lim_{x \rightarrow x_0} f(x) = L$$

• :_____

$$\lim_{x \rightarrow x_0} f(x) = -\infty \Rightarrow \lim_{x \rightarrow x_0} \sqrt[n]{f(x)} = +\infty$$

• :_____

$$f(x) = \sqrt[3]{x^4 - 16} : \quad D_f$$

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• :_____

أ- D_f

• :_____

ب- (1): $\lim_{x \rightarrow 0} \frac{x}{2 - \sqrt[3]{x} + 8}$ (2): $\lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{\sqrt[3]{x} + 56 - 4}$ (3): $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1+x}}{1 - \sqrt[3]{1+x}}$

• :_____

(4): $\lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - \sqrt[4]{2x}}{x - 8}$ (5): $\lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x}}{\sqrt[3]{8x} + \sqrt[3]{x^2}}$ (6): $\lim_{x \rightarrow +\infty} x - \sqrt[3]{x^3 - x^2}$

• :_____

(7): $\lim_{x \rightarrow -\infty} x + \sqrt[4]{1+x^4}$ (8): $\lim_{x \rightarrow +\infty} \sqrt[12]{x} \frac{\sqrt{1+x} - \sqrt[4]{x}}{\sqrt[3]{1+x} - \sqrt[3]{x}}$